Analysis on Docking Process in Lattice-type Self-reconfigurable

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Abstract

Lattice-type self-reconfigurable modular robot consists of many identical modules. It is a complicated nonlinear system with multi-degree of freedom. By changing the connections among modules, the whole configuration of the robot can transform into arbitrary other nonlinear configurations. The docking process is analyzed with the geometric method. The docking force between two neighbouring modules is described to finish the connections.

Keywords: Self-reconfigurable; Docking; Robot

1. Introduction

Self-reconfigurable modular robot (SMR) can dynamically change its geometric structure to complete different requirements of various tasks and environment. It is a complicated nonlinear system with multi-degree of freedom. The SMR can be classified as the lattice-type and the chain-type. The chain-type, such as Polybot \cite{1}, CONRO \cite{2}, has a higher degree of mobility than the lattice-type does. The lattice-type, on the other hand, can easily self-reconfigure and is suitable for forming various static configurations, but it has difficulty in generating motion, such as a cubic structure \cite{3}, Fracta 3D \cite{4}, ATRON \cite{5}, M-TRAN \cite{6} and M-Cube \cite{7}.

In this paper, the docking process in the lattice-type self-reconfigurable robot (M-Cube) is analyzed with the geometric method. The docking force between two modules is described to finish the connections.

2. Analysis of the docking process

The lattice-type self-reconfigurable robot (M-Cube) is a nonlinear system with multi-degree of freedom. As more modules are added to the system, the possible reconfiguration of a group of modules from an initial configuration to other configuration based on some constraints increases exponentially. However, docking is a crucial action for M-Cube to finish different configurations. In fact, the docking process of M-Cube is that one peg of module $i$ inserts one hole of module $j$, and at the same time one peg of module $j$ inserts one hole of module $i$. It is a complicated multiple peg-in-hole process.

2.1 The states of docking and constraint

In the paper, two dimensional problems are discussed. The geometric models of one peg and one hole in module $i$ and module $j$ are shown in Fig.1. We make the assumption: there exits tilt angle $\theta$ between two modules. The boundary state of docking is shown in Fig. 2. Its geometric constraint is

\begin{equation}
\begin{cases}
h_0 s \theta_0 + 2r_p c \theta_0 = 2r_{hi} \\
h_n c \theta_n = 2r_n s \theta_n
\end{cases}
\end{equation}

When $\theta$ is more than $\theta_0$, two modules cannot align. Pegs cannot insert into holes. They cannot finish the docking process. Thus, the tilt...
angle must be adjusted to make $\theta << \theta_0$ and avoid sticking. When $\theta < \theta_0$, because the uncertainty of geometry and control, contact states exist during two modules docking. From the geometry constraints, we know there are at most four-point contact states. The geometric constraint of two-point contact state in the left peg and the left hole in Fig.2 is

$$h_{ij} s\theta + 2r_p c\theta = 2r_H,$$

(2)

The dimensions of two modules are the same. We can obtain $r_p = r_P, r_H = r_H$. Thus, $h_{ii} = h_{ji}$, there are two-point contact states in the right peg and the right hole. Thus, there are at most four-point contact states in two modules docking. The pose of the motion module should be adjusted and the tilt angle should be reduced to avoid appearing contact states. Where $h_{ii}$ is an insertion depth of the left peg (module $M_i$) into the left hole (module $M_i$); $h_{ji}$ is an insertion depth of the right peg (module $M_j$) into the right hole (module $M_j$). The peg of module $M_i$ has a radius of $r_p$, whereas the peg of module $M_j$ has a radius of $r_p$. The radii of one hole in module $M_i$ and one hole in module $M_j$ are $r_{Hi}$ and $r_{Hj}$, respectively. $\theta$ and $\theta_0$ are angles between the axes of a peg and a hole. $D_i, \nu_i$ represent the distance between the peg’s axis and the hole’s axis of module $M_i$, module $M_j$, respectively.

2.2 Contact force analysis of docking

The coordinate system $(oxyz)$ is set up at the center of each module in Fig.1. We can obtain the vectors $r_{i}$ of the contact points relative to the origin $o$ in the coordinate system $oxyz$. Where $g$ represents all contact states, $g=1,2,3,4$. In two dimensions, the motion module rotates $\theta$ about the $x$-axis and translates at $p_x, p_y, p_z$ along the $x$-axis, $y$-axis, $z$-axis. The transformation matrix is

$$T = \begin{bmatrix}
  c\theta & 0 & s\theta & p_x \\
  0 & 1 & 0 & p_y \\
 -s\theta & 0 & c\theta & p_z \\
  0 & 0 & 0 & 1
\end{bmatrix}$$

(4)

The wrenches of the contact cases are described as follows

$$\hat{\mathbf{F}}_i = \mathbf{F}_i + \varepsilon (\mathbf{F}_i \times \mathbf{r}_i)$$

(5)

$\mathbf{F}_i$ is $\hat{\mathbf{F}}_i$ or $\mathbf{f}_i$, $\mathbf{f}_i = \mu \mathbf{F}_i$. $\hat{\mathbf{F}}_i$ is the wrench of the contact force; $\mathbf{f}_i$ is the wrench of the
friction force corresponding to the contact force. The real part of the above equation (5) expresses the contact forces and the dual part expresses the moments. Thus, we can obtain

\[
\hat{\mathbf{F}}_i (\vec{F}_u, \vec{F}_v, \vec{F}_w, (\vec{F}_i \times r_i)_x, (\vec{F}_i \times r_i)_y, (\vec{F}_i \times r_i)_z)
\]

According to Fig.1, we can obtain:

\[
\mathbf{F}_q : (F_q \ 0 \ 0) \quad (6)
\]

\[
f_q : (0 \ 0 \ f_q) \quad (7)
\]

\[
\mathbf{F}_m : T^{-1}(F_m \ 0 \ 0) \quad (8)
\]

\[
f_m : T^{-1}(0 \ 0 \ f_m) \quad (9)
\]

\[
\vec{M}_g = \vec{F}_r \times r_g \quad (10)
\]

where \(q\) represents the docking state, \(q=1,3; \ m\) represents the docking state, \(m=2,4\).

The wrenches of the forces are as follows,

\[
(F_x \ 0 \ 0 \ M_x \ 0 \ 0, 0 \ F_y \ 0 \ 0 \ M_y, 0 \ 0 \ F_z \ 0 \ 0 \ M_z) \quad (11)
\]

Here \(F_x, F_y, F_z, M_x, M_y, M_z\) represent measured forces and moments in the coordinate system(\(xyz\)).

The application of the static equilibrium condition results to the following equations,

\[
\sum (F_{gx} + f_{gx}) + F_x = 0 \quad (12)
\]

\[
\sum (F_{gy} + f_{gy}) + F_y = 0 \quad (13)
\]

\[
\sum (F_{gz} + f_{gz}) + F_z = 0 \quad (14)
\]

\[
\sum (M_{gx} + M_{gy}) + M_{gz} = 0 \quad (15)
\]

\[
\sum (M_{gx} + M_{gy}) + M_{gz} = 0 \quad (16)
\]

\[
\sum (M_{gx} + M_{gy}) + M_{gz} = 0 \quad (17)
\]

Thus, the contact force can be known. From the above analysis, we know that the docking process of two modules is a complicated multiple peg-in-hole process. According to the force/moment, the pose of a motion module and the driven force can be adjusted to make two modules align and finish docking.

3. The locking force of two modules’ docking

After two modules align and finish docking, the worst part of force is two docking rotating arms. As a rotating arm has a peg and a connecting hole, we make the locking force analysis of the vertical connections. Figure 3 shows the connection of single peg and single hole after the peg-in-hole action has been finished. Figure 4 shows the force analysis of the pin, the bead and the hole. Since figure 4 gives the condition of only one bead with the hole and the pin, we make the force triple. Here \(G\) stands for the gravity of the module, \(\beta\) stands for the chamfering of the pin, \(N\) stands for the pin's pressure from one bead, \(N'\) stands for one bead's pressure from the pin, we can obtain

\[
N' = N \quad (18)
\]

![Fig. 3: Connection of peg and hole.](image-url)

\(F_{pull}\) stands for the pressure on the pin, \(F_{support}\) stands for the bearing capacity of one bead, \(F_{bead}\) stands for the resultant force of the pressure on the hole from one bead, \(\alpha\) stands for the angle between the direction of the horizontal axis and that of the resultant force, \(F'_{bead}\) stands for the resultant force of the bead from the pressure of the hole.
Fig. 4: Force diagram of the pin, ball and hole.

So we can obtain

$$F'_{\text{bead}} = F_{\text{bead}} \quad (19)$$

$$F_{\text{pull}} = 3N \sin \beta \quad (20)$$

$$F_{\text{support}} + N' \sin \beta = F_{\text{bead}} \sin \alpha \quad (21)$$

$$F_{\text{bead}} \cos \alpha = N' \cos \beta \quad (22)$$

$$3F_{\text{bead}} \sin \alpha = \frac{1}{2} G \quad (23)$$

From the equations (18) to (23), we can obtain

$$F_{\text{pull}} = \frac{\tan \beta}{2 \tan \alpha} G, \quad (0 < \alpha < \frac{\pi}{2}) \quad (24)$$

From equation (24), when $G$ is constant, $\alpha$ is maximum, $F_{\text{pull}}$ is minimum pull force on the pin. From equation (21), the vector sum of $N'$ and $F_{\text{support}}$ is $F_{\text{bead}}$. It shows that $N'$ of the connection is also used to balance $G$. For this connection, when $G$ increases, both $F_{\text{pull}}$ and $F'_{\text{bead}}$ increase, $F_{\text{bead}}$ also increase, but due to the cooperation of $F_{\text{support}}$ and $N'$ for balance, the reliability of the connection will increase when $F_{\text{pull}}$ increase, and the module's weight $G$ increase or interference make the force that the mechanism bears increases.

4. Conclusion

M-Cube is a complicated nonlinear system with multi-degree of freedom. The states of docking and constraint between two modules are analyzed with the geometric method and the contact force and locking force of docking are described to finish the connections and shape different configurations.

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References


