UKF based Robust Attitude Control for Helicopter

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Abstract - In order to handle the model uncertainty and the external disturbance, a robust attitude control method for helicopter robots is proposed in this paper. Unscented Kalman Filter (UKF) and backstepping technique are adopted in the attitude control design. Model-based backstepping control is presented to keep the desired helicopter attitude. UKF is employed for online estimation of both motion states and model errors of the helicopter. Such estimation results are further incorporated into the controller of helicopter. The backstepping control enhanced by backstepping control is degraded by model uncertainties and time-varying unknown perturbations.

In this paper, an UKF based backstepping control is proposed to control the unmanned helicopter attitude. The UKF is used to estimate the model error or external disturbance in real time. The active model is further integrated into the normal backstepping control to enhance with the ability of reconﬁguring itself adaptively according to the current model errors information. Extensive simulations are conducted with respect to the dynamics of a helicopter to verify the proposed scheme.

II. HELICOPTER ATTITUDE DYNAMIC

Assuming that the flight positions and velocities along the x-, y- and z-axes are very small, the attitude dynamics of the helicopter can be derived from the six degrees of freedom. The differential equation governing the attitude dynamics of the helicopter can be derived from the six degrees of freedom.

\[
\begin{align*}
\dot{\phi} &= p + q \sin \phi \tan \theta + r \cos \phi \tan \theta \\
\dot{\theta} &= q \cos \phi - r \sin \phi \\
\dot{\psi} &= q \sin \phi \sec \theta + r \cos \phi \sec \theta \\
\dot{\rho} &= qr \left( I_\rho - I_z \right) / I_\rho + L / I_z \\
\dot{\iota} &= pr \left( I_\iota - I_\rho \right) / I_\iota + M / I_z \\
\dot{\tau} &= pq \left( I_\tau - I_\rho \right) / I_\tau + N / I_z
\end{align*}
\]

where \( p, q \) and \( r \) are the fuselage coordination system angular velocity components; \( \phi, \theta \) and \( \psi \) are Euler angles, that is, fuselage attitude angles; \( I_x, I_y \) and \( I_z \) are the inertia moments of the helicopter; \( L, M \) and \( N \) are aerodynamic moments about the centre of gravity and they can be expressed as

\[
L = S_{11} b_1 + S_{21} Q_M, M = S_{31} a_1 + S_{32} T_M + S_{33} Q_T, \\
N = S_{41} Q_M + S_{42} T_T, T_T = S_{51} a_1 + S_{52} T_M + T_T = S_{61} a_1 + S_{62} T_M + S_{71} a_1 + S_{72} T_M + S_{73} Q_T,
\]

where \( a_1 \) and \( b_1 \) are the longitudinal and lateral inclination of the main rotor; \( S_{11}, S_{21}, S_{31}, S_{32}, S_{33}, S_{41}, S_{42}, S_{51}, S_{52}, S_{53}, S_{61}, S_{62}, S_{63}, S_{71}, S_{72}, S_{73}, S_{81}, S_{82}, S_{83}, S_{91}, S_{92}, S_{93} \) are physical parameters; \( T_T \) and \( Q_T \) are the tail rotor force and torque; \( T_M \) and \( Q_M \) are the thrusts and torque generated by the main rotor; \( \theta_T \) and \( \theta_M \) are...
the collective pitches of the main and tail rotors, respectively. In this paper, \( \theta_m \) is assumed to be a constant.

By integrating model uncertainty and external disturbance, the system model of (1) can be written as:

\[
\begin{align*}
\dot{x}_1 &= f(x_1)x_2 \\
\dot{x}_2 &= f(x_2) + Gu + C + \Delta f \\
y &= h(x_1, x_2)
\end{align*}
\]

where

\[
x_1 = \begin{bmatrix} \phi \\ \theta \\ \psi \\ \varphi \\ p \\ q \\ r \end{bmatrix}, \quad x_2 = \begin{bmatrix} \phi \\ \theta \\ \psi \\ \varphi \\ p \\ q \\ r \end{bmatrix}, \quad u = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}, \quad \Delta f = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix},
\]

\[
C = \begin{bmatrix} S_{1L}Q_\alpha / I_x \\ (S_{1I}T_\alpha + S_{1L}S_{Q_\alpha} / I_x) \\ (S_{1I}T_\alpha + S_{1L}S_{Q_\alpha} / I_x) \end{bmatrix}, \quad f_i(x_2) = \frac{q\nu(I_y - I_z)/I_x}{pr(I_z - I_x)/I_y}, \frac{pq(I_x - I_z)/I_z}{I_x}
\]

\[
J(x_1) = \begin{bmatrix} 0 & 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & \sin \phi \sec \theta & \cos \phi \sec \theta \\ \end{bmatrix}, \quad
\]

\[
G = \begin{bmatrix} S_{1L}/I_x & 0 & S_{M3}S_{Q1}/I_y \\ 0 & 0 & S_{TT}S_{N2}/I_z \end{bmatrix}
\]

\[
h(x_1, x_2) = \begin{bmatrix} \phi \\ \theta \\ \psi \\ \varphi \\ p \\ q \\ r \end{bmatrix}
\]

\( \Delta f \) is the system external disturbances or the uncertainty and \( y \) is the system output.

### III. UKF-based Joint Estimation

#### A. UKF of nonlinear system

Instead of propagating the Gaussian variables through the first-order linearization of nonlinear model as EKF does, the UKF uses the nonlinear model directly by means of Unscented Transformation (UT). The UT approximates the state distribution with a finite set of points, named sigma points (shown in Fig.1). The sigma points are calculated from the \( \text{a priori} \) mean and covariance.

Let \( x \) denote a n-dimension stochastic variable with the mean of \( \bar{x} \) and the covariance of \( P \). The UT propagates \( x \) through a nonlinear function \( y = f(x) \) to calculate the mean (\( \bar{y} \)) and covariance (\( P \)) of the output \( y \). The UT includes the following steps (also shown in Fig.1).

**Definition of Sigma Points:** The distribution of \( x \) is approximated by a finite set of points named sigma points. The sigma points are calculated from the \( \text{a priori} \) mean and covariance of \( x \) using:

\[
\begin{align*}
\chi_0 &= \bar{x} \\
\chi_i &= \bar{x} + (\sqrt{(I + \lambda)} P_{x})_{i} \quad i = 1, \ldots, n \\
\chi_i &= \bar{x} - (\sqrt{(I + \lambda)} P_{x})_{i-I} \quad i = l+1, \ldots, 2n
\end{align*}
\]

where \( (\cdot)_{i} \) is the \( i \)th row of matrix \( (\cdot) \) and \( \lambda = L(\alpha^2 - 1) \) is a scaling parameter.

**Spread of Sigma Points:**

\[
\gamma_i = f(\chi_i) \quad i = 1, \ldots, 2n
\]

**Calculation of the Mean and Covariance of \( y \)**

\[
\begin{align*}
\bar{y} &= \sum_{i=0}^{2n} w_{i}^{m} \gamma_i \\
P_y &= \sum_{i=0}^{2n} w_{i}^{c} (\gamma_i - \bar{y})(\gamma_i - \bar{y})^T
\end{align*}
\]

where the weights of \( w_{i}^{m} \) and \( w_{i}^{c} \) are calculated using

\[
w_{0}^{m} = \frac{\lambda}{I + \lambda} \quad w_{i}^{c} = \frac{\lambda}{I + \lambda} + (1 - \alpha^2 + \beta) \quad w_{i}^{m} = \frac{1}{2(I + \lambda)} \quad i = 1, \ldots, 2n
\]

The constant \( \alpha \) in (8) determines the spread of the sigma points. Constant \( \beta \) is used to incorporate part of the prior knowledge of the distribution of \( x \) (for Gaussian distributions, \( \beta = 2 \) is optimal).

The UKF is a straightforward extension of the UT approach to the recursive estimation. Consider the general discrete nonlinear system:

\[
\begin{align*}
\bar{x}_k &= f(\bar{x}_{k-1}, u_k) + w_k \\
\bar{y}_k &= h(\bar{x}_k) + v_k
\end{align*}
\]

where \( \bar{x}_k \in R^n \) is the state vector, \( u_k \in R^l \) is the input vector, \( \bar{y}_k \in R^m \) is the output vector at time \( k \). \( w_k \) and \( v_k \) are, respectively, the disturbance and sensor noise vector, which are assumed to Gaussian white noise with zero mean. The UKF for state estimation is given as follows.

**Initialization**

\[
\begin{align*}
\bar{x}_0 &= E[x_0] \\
P_0 &= E[(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T]
\end{align*}
\]
Since the dynamics of $\Delta f^e_k$ is unknown, it is assumed as a non-correlated random drift vector, and $w_{bk}$ is the Gaussian white noise with zero mean.

In UKF-based joint estimation, an augmented vector is defined as a combination of state and parameter to be estimated, i.e., $x^a_k = [x_k, \Delta f^e_k]^T$, then (13) can be rewritten as:

$$
\begin{align*}
x^a_k &= \tilde{f}(x^a_{k-1}, u_k) + w_k^a \\
y_k &= \tilde{h}(x^a_k) + v_k 
\end{align*}
$$

The UKF-based state estimation introduced in Section A can then be applied to this augment equation. It should be pointed out that even in linear system the state and parameter joint estimation might be nonlinear.

IV. UKF-BASED BACKSTEPPING CONTROL

A. Classic Backstepping Control

To develop the model-based attitude control, we define error variables $z_1 = x_1 - x_{id}$ and $z_2 = x_2 - \alpha$, where $\alpha$ is virtual control law. Then we have

$$
\begin{align*}
\dot{z}_1 &= \dot{x}_1 - \dot{x}_{id} = J(x_1)x_2 - \dot{x}_{id} = J(x_1)(z_2 + \alpha) - \dot{x}_{id} \\
\dot{z}_2 &= \dot{x}_2 - \dot{\alpha} = f(x_2) + Gu + C + \Delta f - \dot{\alpha}
\end{align*}
$$

Choose the virtual control law $\alpha$ as

$$
\alpha = J^{-1}(x_1)(-k^T_1z_1 + \dot{x}_{id})
$$

where $k^T_1 > 0$. The time derivative of $\alpha$ is

$$
\dot{\alpha} = J^{-1}(x_1)(-k^T_1z_1 + \dot{x}_{id}) + J^{-1}(x_1)(-k^T_1z_1 + \dot{x}_{id})
$$

Substituting (15) into (14), we obtain

$$
\begin{align*}
\dot{z}_1 &= \dot{x}_1 - \dot{x}_{id} = J(x_1)x_2 - \dot{x}_{id} = J(x_1)(z_2 + \alpha) - \dot{x}_{id} \\
\dot{z}_2 &= \dot{x}_2 - \dot{\alpha} = f(x_2) + Gu + C + \Delta f - \dot{\alpha}
\end{align*}
$$

Consider the Lyapunov function candidate

$$
V_1 = \frac{1}{2}z_1^T z_1
$$

The time derivative of $V_1$ is

$$
\dot{V}_1 = z_1^T \dot{z}_1 = -z_1^T k_1 z_1 + z_1^T J(x_1) z_2
$$

The first term on the right-hand side is negative. Next what we will do is to cancelled the second term.

Consider the Lyapunov function candidate

$$
V_2 = V_1 + \frac{1}{2}z_2^T z_2
$$

Invoking (20), the time derivative of $V_2$ is

$$
\dot{V}_2 = \dot{V}_1 + z_2^T \dot{z}_2 = -z_2^T k_2 z_2 + z_2^T J(x_1) z_2
$$

The input control $u$ is proposed as follows

$$
u = G^{-1}(\dot{\alpha} - J^{-1}(x_1) z_1 - f(x_2) + Gu + C + \Delta f - \dot{\alpha})
$$

If $\Delta f$ is known, then the closed-loop system is stable. But in most situations the model uncertainty and external disturbances are unknown. Then the control performance will be degraded, and even will lead to system unstable. In the next
section, we will solve the problem.

### B. Enhanced Backstepping Control

The structure of the enhanced backstepping control is shown in Fig.2. The outputs of UKF-based estimation introduced in section 3 are fed to the controller. The ‘noise-free’ states and the introduced in section 3 are fed to the controller. The dynamics of an unmanned helicopter.

![Enhanced backstepping control](image)

#### V. SIMULATIONS

In this section, the proposed UKF-based enhanced backstepping control algorithm is verified with respect to the dynamics of an unmanned helicopter.

#### A. Model for Simulations

The helicopter attitude model of (1) is used in our simulation, which is described by [10]. The parameters in the attitude dynamics of (1) are selected as Table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value in Simulation</th>
<th>Symbol</th>
<th>Value in Simulation</th>
</tr>
</thead>
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<tr>
<td>$I_x$</td>
<td>0.1634 kgm$^2$</td>
<td>$S_{NN}$</td>
<td>0.9</td>
</tr>
<tr>
<td>$I_y$</td>
<td>0.5782 kgm$^2$</td>
<td>$S_{TT}$</td>
<td>1777</td>
</tr>
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<td>$I_z$</td>
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<td>$S_{MM}$</td>
<td>39.8</td>
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<td>$S_{11}$</td>
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<td>$S_{T1T1}$</td>
<td>106.2</td>
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<td>$S_{12}$</td>
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<td>95.6</td>
</tr>
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</tr>
<tr>
<td>$S_{33}$</td>
<td>-1</td>
<td>$S_{T2T2}$</td>
<td>-0.03</td>
</tr>
</tbody>
</table>

#### A. The UKF Settings

The augmented state and the measurement vector are respectively selected as:

\[
x = \begin{bmatrix} \phi, \theta, \psi, p, q, r, d_1, d_2, d_3 \end{bmatrix}^T
\]

\[
y = \begin{bmatrix} \phi, \theta, \psi, p, q, r \end{bmatrix}^T
\]

Then (3) can be rewritten in a nonlinear state-space form by the definition of (23).

The initial state is selected as $x_{t_0} = \hat{0}$, and sampling interval is $\tau = 0.01$s. The attitude states and measurements are corrupted by zero mean additive white noise with covariance

\[
Q^* = \text{diag} \begin{bmatrix} 10^{-8}, 10^{-8}, 10^{-8}, 10^{-8}, 10^{-8} \end{bmatrix}
\]

\[
Q^*_k = \text{diag} \begin{bmatrix} 10^{-4}, 10^{-4}, 10^{-4}, 10^{-4}, 10^{-4} \end{bmatrix}
\]

The UKF parameters are set as:

\[
\begin{bmatrix} P_0, Q, R \end{bmatrix} = \begin{bmatrix} Q^*, 0_{6\times6}, Q^* \end{bmatrix}
\]

\[
Q^*_k = \text{diag} \begin{bmatrix} 10^{-7}, 10^{-5}, 10^{-3} \end{bmatrix}
\]

\[
Q = Q^*_k + \beta Q^*_k + \alpha Q^*_k
\]

Where $Q^*_k$ is the covariance of the noise $w$ in Eq. (13). Suitable selection of $Q^*_k$ can achieve good estimation performance of the $\Delta f$.

#### C. Simulation Results and Performance Evaluation

The UKF-based joint estimation of both state and the model errors (or disturbance), as well as the UKF-based backstepping control, are tested.

To evaluate the performance of the control scheme of Fig.2, model error is assumed:

\[
\begin{cases}
    d_1 = d_2 = d_3 = 0 & 0 \leq t < 3s \\
    d_1 = 20, d_2 = 20, d_3 = 20 & t \geq 3s
\end{cases}
\]

Observe the performance of the system for tracking the reference trajectory:

\[
\begin{cases}
    \phi = \theta = \psi = 1 & 0 \leq t < 5s \\
    \phi = \theta = \psi = 0 & t \geq 5s
\end{cases}
\]

The variation of model errors are assumed to be totally unknown to the estimator, so the noise-driven UKF like (13) is used for the estimation.

The estimated model errors and also their actual values are shown in Fig.3. It can see that the UKF gives quite satisfactory results. When the model errors changing, the UKF reacts quickly and tracks the rapid changes successfully.

Fig.4 illustrates the attitude outputs while model errors existing, with respect to the backstepping control with and without UKF-enhancement. We can see that, the tracking errors of classic backstepping control (without UKF estimation) are much more significant than those by the backstepping control enhanced by UKF. This is due to the fact that the reference model of classic backstepping control is totally unconscious to the existence of model errors. As for enhanced backstepping control, the tracking errors due to the model errors are quickly overcome by the active parameter updates involved in the reference model, and the tracking performance is almost the same after a short period (~ 1 seconds) of adaptation.

#### VI. CONCLUSION

The UKF-based joint estimation is used for online modeling both the motion state and model errors of a helicopter. The active model is further incorporated into the classic backstepping control. Simulations on the attitude dynamics of
helicopter explicitly demonstrate the performance of UKF in online tracking the step-changed model error. The backstepping control enhanced by UKF shows clear improvement in attitude tracking performance when being compared with the normal backstepping control based on nominal dynamics.

REFERENCES


