Task-based structure synthesis of source metamorphic mechanisms and constrained forms of metamorphic joints

Shujun Li a,b,⁎, Hongguang Wang b, Qiaoling Meng c, Jian Sheng Dai d

a School of Mechanical Engineering and Automation, Northeastern University, Shenyang 110004, China
b State Key Laboratory of Robotics of China, Shenyang Institute of Automation, Chinese Academy of Sciences, Shenyang 110016, China
c School of Medical Instrument and Food Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China
d King’s College, University of London, Strand, London WC2R 2LS, UK

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ABSTRACT
One of the key works in applied metamorphic mechanism design is to obtain the source metamorphic mechanism and the constraint forms and structures of metamorphic joints to carry out given working tasks. The metamorphic cyclogram and the types and combinations of the constraint metamorphic joints for the tasks are presented according to the task-based kinematic rules. The augmented Assur groups are further classified based on the output movement combinations so that the corresponding relationships between the task-based kinematic requirements and the structures of the augmented Assur groups are established, thereby providing a simple way to the synthesis of topological structures of source metamorphic mechanisms based on the augmented Assur groups. The equivalent resistance matrix of the metamorphic joints is introduced from the metamorphic cyclogram and equivalent resistance gradient of the metamorphic joints. The constraint structure matrix is then obtained using the resistance matrix and the constraint forms and structures of the metamorphic joints. The synthesis method of metamorphic mechanisms with the constraint forms and structures of metamorphic joints is described in steps. The task-based design approaches are demonstrated for a reposition metamorphic mechanism for broken strands of the repair robot used in extra-high-voltage power transmission lines. The results show that proposed design theory and method are feasible and practical.

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1. Introduction

Researches on metamorphic mechanisms have been made in a great progress since it was proposed in 1998 [1]. For instance, PARISE et al. derived the metamorphic Ortho-planar mechanisms in 2000 [2]. The structure synthesis method of metamorphic mechanisms was developed in [3] by matrix operation of the configuration transformations. Liu et al. proposed a new metamorphic way by utilizing the coupled links, the linkage relationship, and the characteristics of their kinematic pairs based on the research results on metamorphic ways of metamorphic mechanisms [4]. Reference [5] discussed the constitution and description ways of metamorphic mechanisms and built a series of metamorphic equations based on the basic characteristics of metamorphic mechanisms. Ding et al. proposed a new design concept of metamorphic mechanisms according to the research results on topological and metamorphic characteristics of metamorphic mechanisms with the identical links, symmetrical constructions, and assembly [6]. Li et al.[7] further presented the Joint-gene based variable topological representations and configuration transformations. They also developed the concept of Assur groups and proposed a structural theory of metamorphic mechanisms based on the augmented Assur group [8,9]. Zhang et al.

⁎ Corresponding author at: School of Mechanical Engineering and Automation, Northeastern University, Shenyang 110004, China. Tel./fax: +86 24 83674338(o).
E-mail address: shjunli@mail.neu.edu.cn (S. Li).
The task required output motion and working status of metamorphic joints

2.1. The task required output motion and working status of metamorphic joints

The type and the type combinations of metamorphic joints should be selected at the first step of the design according to the required motion forms of output links to perform the working task. The working statuses of constrained metamorphic joints will transform between the moving and static modes depending on the change of constrained forces to form the corresponding working stages, while the driver joint and general joints keep working. In order to obtain the corresponding motion forms based on working task, i.e. to obtain the types of kinematic pairs and combination forms, the cyclogram of metamorphic mechanism is introduced. Such this, the corresponding working configuration to perform the intended task (output motion) will be built. The horizontal axis expresses the
change of the metamorphic configuration with the movement of driver link $\theta$, and the vertical axis expresses the relative motion of each kinematic joint $J$. Therefore, the change of configuration and motion of the metamorphic kinematic joints can be described in Fig. 1, which is called the cyclogram of the metamorphic mechanism.

2.2. Motion forms and type combinations of metamorphic joints

Fig. 1 only shows the motion status of each metamorphic joint. However, the type of each metamorphic joint, R (revolute) joint or P (prismatic) joint, should be selected according to the motion forms of output links when we design a metamorphic mechanism. The motion forms of output links and the type combinations of metamorphic joints are shown in Fig. 2 for 2DOF metamorphic mechanisms.

Fig. 2(a) shows that the two metamorphic joints are two R joints followed by the two P joints as shown in Fig. 2(b). Fig. 2(c) and (d) shows that the two metamorphic joints are one R joint and one P joint, respectively.

Compared to the cyclograms shown in Fig. 2, the mobility-configuration matrix $M$ of the 2DOF metamorphic mechanism can be written as:

$$
M = \begin{bmatrix}
0 & J_{12} \\
J_{11} & 0
\end{bmatrix}
$$

where, the working configurations are shown in columns of the matrix and constrained status of metamorphic joints in different working configurations are shown in rows. Herein, “0” means that the metamorphic joint is in a static mode, and $J_{ij} \neq 0$ means that the metamorphic joint is in the movement (working) mode.

3. Structural synthesis of source of metamorphic mechanisms based on motion forms provided by the groups

The synthesis of mechanism, based on the Assur groups and augmented Assur groups, is classified according to their structures so far. However, the synthesis of mechanisms based on work task in practice is the demanding one for structure and output motion forms of the links provided to perform the intended task. Therefore, the augmented Assur groups can be classified further according to the motion forms they can provide. Thus, the relationships among the augmented Assur group, the motion forms of the output links and the type combinations of metamorphic joints can be set up.

3.1. The augmented Assur groups

According to the work in [8] and [9], if an additional binary link and a R/P joint are inserted into the class II Assur group, the mobility of an augmented group becomes one instead of the zero of class II Assur group, and this is called class II augmented Assur groups in the paper. The link forms of a 1DOF augmented Assur group can be shown in Fig. 3. The two kinds of general driven forms of the driving link are shown in Fig. 4.

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Fig. 2. The motion forms and the type combinations of metamorphic joints.
3.2. The metamorphic groups classified on the output motion forms of augmented Assur groups

3.2.1. Metamorphic group providing 2R output

According to the motion forms and type combinations shown in Fig. 2(a), the seven types of metamorphic groups (i.e. augmented Assur groups) with two R joints from Fig. 3 are selected as shown in Fig. 5. Among these groups, there are four non-symmetry link groups which have two connecting configurations by taking into account the connecting order with the driving links. Therefore, there are eleven connecting configurations in the 2R metamorphic groups.

3.2.2. Metamorphic group providing 2P output

According to the motion forms and type combinations shown in Fig. 2(b), the 6 types' metamorphic groups with 2 prismatic joints from Fig. 3 are selected as shown in Fig. 6. Among these groups, there are four non-symmetry link groups which own 2 connecting configurations by taken into account the connecting order with the driving links. Therefore, there are 10 connecting configurations in 2P metamorphic groups.

3.2.3. Metamorphic group providing 1R–1P output

According to the motion forms and type combinations shown in Fig. 2(c) and (d), the 7 types' metamorphic groups with 1 revolute joint and 1 prismatic joint from Fig. 3 are selected as shown in Fig. 7. Among these groups, there are six non-symmetry link groups which own 2 connecting configurations by taken into account the connecting order with the driving links. Therefore, there are 13 connecting configurations in 1R–1P metamorphic groups.

3.3. Task-based structural synthesis

A source metamorphic mechanism consists of driving links, bases, and the augmented Assur groups according to the constitution theory of mechanisms [8,9]. The process of structural synthesis of a task-based source metamorphic mechanism based on motion forms and type combinations of metamorphic joints is described as follows.

(1) Determining the motion forms and the type combinations of metamorphic joints according to the given working task. The metamorphic cyclogram is obtained based on the task technology and motion demands. The corresponding motion forms and type combinations of metamorphic joints of the corresponding working configurations can be obtained by considering the outputting motion forms of each configuration of the metamorphic joints of the mechanism.

Fig. 3. Class II augmented Assur groups.

Fig. 4. The driving links and frame.
(2) Choosing the corresponding augmented Assur group based on the motion forms and the type combinations of metamorphic joints. First of all, choosing the metamorphic group from the classified augmented Assur groups according to the provided output motions of the group.

(3) Selecting one of the driving forms of Fig. 4, and connecting the driver and the augmented Assur group(s) to the ground to form the structure of the source metamorphic mechanism.

(4) Arranging the order of metamorphic joints according to the outputting techniques of the working task. For instance, the order of metamorphic joints of a 2DOF metamorphic mechanism can be designed as:

① two adjacent metamorphic joints
② two metamorphic joints at the proximal and the distal ends
③ two metamorphic joints in the middle
④ two metamorphic joints anywhere

Finally, one can choose one of cases by considering the technological characteristics and the real working situation. This structural synthesis method can simplify the process of structural synthesis of the metamorphic mechanisms, and isomorphism detecting process.

4. The equivalent resistance matrix of metamorphic mechanisms

4.1. Equivalent resistance gradient model of metamorphic mechanisms

Because a constrained metamorphic mechanism is a kind of under-actuated (or single-driver) mechanism, there is only one metamorphic joint keeps moving (working) at a time in corresponding working stage when the other(s) keeps static in the metamorphic process to prevent the random output motion. The transformations from moving to static of metamorphic joints are controlled by the constraints or constrained forces by the designed structure of metamorphic joints. A new idea of equivalent resistance coefficient is proposed to describe the working statuses and constraint resistance characteristics of the joints, also to compare the constrained forces between revolute and prismatic metamorphic joints. This coefficient is defined as the ratio of the force or torque in moving direction provided by the constraint of the metamorphic joint to the force or torque in the moving direction exerted on the metamorphic joint in the working process.

\[
f_e(\theta_i) = \frac{F_c(\theta_i)}{F(\theta_i)} = \frac{T_c(\theta_i)}{T(\theta_i)} \quad i = 1, 2, \ldots, m
\]

where

\[f_e(\theta_i)\] equivalent resistance coefficient of metamorphic joint,
\[\theta_i\] the displacement of driver in corresponding working stages,
\[m\] the number of the working stages,

Fig. 5. 2R output metamorphic group.

Fig. 6. 2P metamorphic group.
$F_c(\theta_i), T_c(\theta_i)$ resistant force and resistant torque in moving direction provided by the constraint of metamorphic joint, $F(\theta_i), T(\theta_i)$ actual force and torque acting on the metamorphic joint in the moving direction during the working process.

$$f_{em}(\theta_i) = \frac{|F_c(\theta_i)|}{F_{min}} = \frac{|T_c(\theta_i)|}{T_{min}} \quad \text{and} \quad f_{es}(\theta_i) = \frac{|F_c(\theta_i)|}{F_{max}} = \frac{|T_c(\theta_i)|}{T_{max}}$$  \(3\)

$F_{min}$ and $T_{min}$ are the minimum force and the moment of the corresponding link configuration of metamorphic joints, respectively. $f_{em}(\theta_i)$ and $f_{es}(\theta_i)$ are the equivalent resistance coefficients of movement and rest, respectively.

The moving sequences of metamorphic joints should be proportional to the equivalent resistant forces according to the law of minimum resistance of kinematics. In the corresponding working stages, the equivalent resistance coefficient of a moving metamorphic joint is smaller than that of the static metamorphic joint, i.e. the equivalent resistance gradient of the metamorphic joints in the working stages of constrained metamorphic process should be,

$$f_{em}(\theta_i) \leq 1 \quad \text{and} \quad f_{es}(\theta_i) \geq 1.$$  \(4\)

The equivalent resistance gradient sketch of a 2DOF metamorphic mechanism can be illustrated in Fig. 8 according to Eq. (4).

### 4.2. Equivalent resistance matrix of metamorphic mechanisms

In order to build the relationship between the motion of metamorphic joint and its constrained force, the equivalent resistance matrix of a 2DOF metamorphic mechanism can be obtained from the Eq.(1) and Fig. 8, and can be written as,

$$F = \begin{bmatrix} 0_{11} & f_{12} \\ f_{i1} & 0_{12} \end{bmatrix}$$  \(5\)
where, the working configurations are shown in columns of the matrix and the constrained status of metamorphic joints in different working configurations are shown in rows. $f_{ij} > 1$ denotes that the $i$th metamorphic joint is constrained in the $j$th working configuration. $0_{ij}$ denotes that $i$th metamorphic joint keeps working in the $j$th working configuration.

![Diagrams of typical constrained structures of metamorphic joints and the characteristics matrix.](image)

Fig. 9. Typical constrained structures of metamorphic joints and the characteristics matrix.
5. Constrained configuration matrix of metamorphic mechanism

5.1. Constraint type of metamorphic joints

Generally, constrained metamorphic operations are implemented by using geometric constraints and/or force constraints of metamorphic joints to overlap two links to one, or to make the metamorphic joints locked. Proposed typical structures of constrained metamorphic joints and the characteristics matrix are shown as in Fig. 9, where 0 denotes non-constraint state of metamorphic joint, and 1 denotes constraint state of metamorphic joint in the matrix.

5.2. Structural topological matrix of metamorphic joints

The constraint type of the metamorphic joints and the transformation law provided by the structures should accord with the variation of the working configuration equivalent resistance gradient. In order to build the connection-ship of constrained structures of metamorphic joints and its equivalent resistance matrixes in corresponding working stages, structural topology matrix \( C \) is built by comparing Fig. 8 and Eq. (5), and is written as,

\[
C = \begin{bmatrix}
J_i & J_j
\end{bmatrix} = \begin{bmatrix}
0 & c_{ij} \\
c_{ij} & 0
\end{bmatrix}
\]

where, the working configurations are presented in columns of the matrix and constrained force transformation law of metamorphic joints in different working configurations are shown in rows. \( c_{ij}(i, j = 1, 2) \) denotes that the \( i \)th metamorphic joint is constrained in the \( j \)th working configuration, and \( 0_{ij} \) denotes that \( i \)th metamorphic joint keeps working in the \( j \)th working configuration. For instance, \( C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \), which expresses that the two constraint types of the metamorphic joint are geometric constraint and spring force constraint through comparing Fig. 9(a) and (b).

6. Examples of structural synthesis of constrained metamorphic mechanisms

The design of broken strands live-line repairing operation for extra-high-voltage (EHV) power transmission lines is introduced as an example in this paper. The design method and procedure for the task-based constrained metamorphic mechanisms are described below.

6.1. Background and demands of task

The EHV power transmission line can be damaged or broken because of electric arc burning and external force as shown in Fig. 10. The incompact broken wire wrapping should be recovered (repositioned) to the initial conditions before carrying out the repairing working task.

The broken strands reposition component is designed as a kind of repositioning nut with rolling element in its inner wall, which sleeves EHV transmission lines and makes the rolling element embed into the gap between adjacent strands. The process of broken strands repositioning is equivalent to the meshing course between the repositioning nut pushed by a mobile robot and strands of EHV power lines, which will accomplish broken strands reposition operation of lines. Owing that EHV transmission lines are closed, the repositioning nut must be made of two cut-open nuts as shown in Fig. 11(a), which would clamp on the live-line and be locked into a united one (Fig. 11(b)), to prevent the cut-open nuts from bulging loosen while the mobile robot pushes them move along the line during the broken strands reposition operation.

According to the background and demands of task, the operation mechanism must implement two tasks. One is to make the cut-open nuts to be fixed on the line. The other is that the pin locks the cut-open nuts. Besides, the cut-open nuts can be unlocked and collected for the next operation after completing the task.

Fig. 10. The broken power transmission line.
6.2. Design of the task-based source metamorphic mechanism

6.2.1. To determine the metamorphic cyclogram

A metamorphic mechanism has the feature of multi-configuration and under-actuation. This kind of mechanism can reduce the weight of robot, simplify the control system, and save the limited carrying energy, thereby meeting the task demands. The metamorphic cyclogram of a 2-DOF metamorphic mechanism which can implement both the demands is shown in Fig. 12.

6.2.2. To obtain the motion of the outputting configuration based on the task

The motion and types of metamorphic joints are obtained according to the technological demands of the task. Then, we simply the structural synthesis region by choosing one of the type combinations of metamorphic joints as shown in Fig. 2. Therefore, the movement mode of each metamorphic configuration can be obtained as follows.

Configuration 1  revolute (swing) motion output to provide the motion of making the cut-open nuts to be fixed on the EHV power line.
Configuration 2  revolute (swing) motion output to provide a revolute driving motion for the cut-open nut locking mechanism.

6.2.3. To choose the type of metamorphic joint based on the output motion

The motion output of the configuration 1 is revolute (swing), and the motion output of the configuration 2 is also revolute (swing). The desired motion output in both the configurations can be obtained when the corresponding metamorphic joint keeps static (constrained metamorphic). Therefore, we choose two R joints as the constraint metamorphic joint for both the configurations as shown in Fig. 2(a).

6.2.4. Design of the source metamorphic mechanism

We need to choose the metamorphic joints in the augmented Assur groups. The two constrained kinematic joints are both needed to handle the cut-open nuts according to the working task. Therefore, the two adjacent metamorphic joints at the end of the metamorphic group are chosen. Based on Section 3.2, there are seven topological structures for the 2R metamorphic joints. There are four topological structures with two adjacent R joints at the end of the groups so that the RRRR metamorphic group is chosen as the metamorphic joints by considering the manufacturing technologies and cost.

In order to meet the metamorphic structural synthesis of metamorphic joints and take into account the reliability of the clamp operation, this work chooses a P joint as driver (i.e., a spiral with reverse self-locked). A source metamorphic mechanism is constructed based on the structure theory of the Assur group as shown in Fig. 13.

6.3. Design of constrained configuration of metamorphic mechanism

If the C and D joints in Fig. 13 are chosen as the constrained metamorphic joints, its equivalent resistance gradient curve is shown in Fig. 14 according to the equivalent resistance gradient model mentioned above.

![Fig. 11. The cut-open nuts.](image)

![Fig. 12. The metamorphic cyclogram.](image)
Therefore, the equivalent resistance gradient curve can be expressed in matrix as follows according to Eq.(5).

\[
F = \begin{bmatrix}
0 & f_{45} \\
f_{34} & 0
\end{bmatrix}
\]  

(7)

Then, the structural topological matrix of the constrained metamorphic joints can be built as follows from Eq.(6).

\[
C = \begin{bmatrix}
R_{45} \\
R_{34}
\end{bmatrix} = \begin{bmatrix}
0 & c_{45} \\
c_{34} & 0
\end{bmatrix}
\]  

(8)

Compared the elements of the matrix and the constrained matrix shown in Fig. 9, the constrained form of constrained metamorphic joints can be obtained as follows.

\[
R_{45} = [g; g_t]; \quad R_{34} = [s; s_t; s_g]
\]  

(9)

Considering the real working conditions, R34 is in a static mode when the cut-open nuts are fixed at the power lines. The joint should be combined with links 3 and 4. Therefore, the metamorphic joints with geometric limit and spring force constraints \( s_g, s_t g \) meet the demands. Link 4 can be considered as a whole with the base when the cut-open nuts are fixed on the lines. Subsequently, the metamorphic joint R45 should be chosen as the geometric constraints \( g \). Substitute the symbols introduced in Fig. 9 to the structural topological matrix of metamorphic joints, the constrained structural topological matrix can be written as,

\[
C = \begin{bmatrix}
0 & c_{45} \\
c_{34} & 0
\end{bmatrix} = \begin{bmatrix}
0 & g \\
s_g/s_t & 0
\end{bmatrix}
\]  

(10)

According to the permutation and combination principle, the matrix \( C \) can be written as,

\[
C_1 = \begin{bmatrix}
0 & g \\
s_g & 0
\end{bmatrix}; \quad C_2 = \begin{bmatrix}
0 & g \\
s_t g & 0
\end{bmatrix}
\]

(11)

Substituting \( C_i \) \((i = 1, 2)\) to the source metamorphic mechanism shown in Fig. 13, the two kinds of metamorphic mechanisms meeting the working tasks are obtained as shown in Fig. 15.

6.4. Design of the reposition constrained metamorphic mechanism

The selection of the structure form is based on the reliable working configuration transforming ability and the structure simplicity. From this point of view, Fig. 15(b) mechanism is selected as the practical broken strands reposition metamorphic mechanism, meaning that metamorphic joint C is with spring force constraint and geometric constraint and the joint D is with geometric constraint.
Considering the symmetrical operation of practical mechanism, finally the practical broken strands reposition metamorphic mechanism designed is shown in Fig. 16(a). The other two ones are the cut-open nuts locking mechanism in x–z plane and reverse un-locked mechanism as shown in Fig. 16(b) and (c), respectively.

6.5. The working principle of the metamorphic mechanism

The working configurations analysis of metamorphic process is elaborated below. In its initial working configuration as shown in Fig. 16(a), link 3 and clamp 4 are fixed together by the spring 10 and a special geometrical constrain.

When the operating starts, screw 5 rotates driven by the motor, and screw nut 1 moves towards the symmetrical center which drives clamp 4 swing around D to carry the cut-open nuts 11 to clamp and tighten the EHV power line 13. After the cut-open nuts’ clamping operation is finished, the clamp 4 will be fixed as a part of the frame. Along with the motion of screw nut 1, link 3 begins to rotate around joint C after overcoming the spring resistance to provide a driving motion for the locking mechanism as shown in Fig. 16(b).

The 3D model and the prototype of the mechanism are shown in Fig. 17(a) and (b), respectively.

In the real situation, the reposition metamorphic mechanism proposed will be carried by a mobile robot and mounted onto the EHV power line. The united cut-open nut engages with the strands of power line to recover the broken lines to the initial position. The disassemble operation of the united cut-open nut will be done by the mechanism when the mobile robot pushes the nut to a position where the nut’s phase angle is 180°relative to the initial orientation.

7. Conclusions

In this paper, the working task has been decomposed and transformed as the outputting motion of a metamorphic mechanism in order to obtain the type combinations of metamorphic joints. The relationship among the working task and the motion and the type combinations of metamorphic joints is built thereby forming the corresponding metamorphic cyclogram.

According to the outputting forms and the linkage types, the augmented Assur groups are classified. The relationship between the working task and the corresponding metamorphic groups can be obtained directly based on the classified metamorphic groups. The structural synthesis and selection of source metamorphic mechanisms are located in the task-matching region so that the synthesis method of source metamorphic mechanisms becomes more practical.

A kind of configuration synthesis methods for metamorphic mechanisms is proposed to derive the corresponding constraint types and constraint structures of metamorphic based on the source metamorphic mechanisms. The relationship between the configuration transformation and the variation of constraint forces of metamorphic joints is obtained by comparing the metamorphic cyclogram and the equivalent resistance curve of metamorphic joints. Also, in order to select the constraint forms and structures of metamorphic
The relationship among the equivalent resistance variations, the constraint forms and the matrix expression is derived through inferring the constraint forms, constraint forces and structures of metamorphic joints.

A new approach for designing task-based metamorphic mechanism has been presented. The effectiveness and practicability of the design theory and method proposed in this paper have been validated by designing the reposition constrained metamorphic mechanism for the EHV power line repairing robot.

It is noted that this paper only focuses on the structural synthesis and design of the 2-DOF constrained metamorphic mechanism. However, the analysis and design of a multi-DOF metamorphic mechanism can be developed based on the work of this paper.

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Fig. 17. The 3D model and the prototype of the mechanism.