A kind of hybrid pneumatic-piezoelectric flexible manipulator system has been presented in the paper. A hybrid driving scheme is achieved by combining of a pneumatic proportional valve based pneumatic drive and a piezoelectric actuator bonded to the flexible beam. The system dynamics models are obtained based on system identification approaches, using the established experimental system. For system identification of the flexible piezoelectric manipulator subsystem, parametric estimation methods are utilized. For the pneumatic driven system, a single global linear model is not accurate enough to describe its dynamics, due to the high nonlinearity of the pneumatic driven system. Therefore, a self-organizing map (SOM) based multi-model system identification approach is used to get multiple local linear models. Then, a SOM based multi-model inverse controller and a variable damping pole-placement controller are applied to the pneumatic drive and piezoelectric actuator, respectively. Experiments on pneumatic driven vibration control, piezoelectric vibration control and hybrid vibration control are conducted, utilized proportional and derivative (PD) control, SOM based multi-model inverse controller, and the variable damping pole-placement controller. Experimental results demonstrate that the investigated control algorithms can improve the vibration control performance of the pneumatic driven flexible piezoelectric manipulator system.

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manipulators, Euler-Bernoulli beams are used [3]. The experimental methods used to obtain the model are based on system identification. Active vibration control employs actuators to utilize external force effects on the mechanical system in order to dissipate energy. There are many well-known traditional actuating components utilized in active vibration control, such as electromagnetic devices, pneumatic actuators, rotary and linear motors etc. The most common smart materials used in active structures are shape memory alloys, magneto- and electrostrictive materials, semi-smart magneto- and electrorheological fluids, electrochemical and piezoelectric materials [4].

Classical feedback methods have been used in active vibration control. A delayed position feedback control is used for single-link flexible manipulator vibration control by Jmfene [5]. Shan et al. [6] analyzed the positive position feedback (PPF) controller and applied to PZT actuators for suppressing multi-mode vibrations. The bang-bang control is used by Tzou and Chai [7] for a hybrid polymeric electrostrictive/piezoelectric beam vibration control. Adaptive input shaping control has been investigated in vibration control [8]. Adaptive phase adjusting controller is proposed by Qiu and Zhao [9] for a pneumatic driven flexible manipulator vibration control system to deal with the long time delay introduced by the actuator.

Pneumatic actuators have the advantages of low cost, high power-to-weight ratio, ease of maintenance, cleanliness, power source being readily available and cheap [10]. The compressibility of air results in very low stiffness (compared with the hydraulic system) leading to low natural frequency. That low damping of the actuator system makes it difficult to control, especially with the presence of nonlinearities, time varying effects and position dependence.

System identification methods fall into two broad categories, i.e., global and local. Global models have shown some difficulties in cases when the dynamical system characteristics vary considerably over the operating regime, effectively bringing the issue of time varying parameters in the design. Nonlinear systems can be handled by linearization around multiple working points. The general idea of the multiple model approach is to represent a nonlinear behavior by a set of linear models that are connected together with the usage of an interpolation mechanism [11]. Each sub-model describes the behavior of the nonlinear system in a limited operating range.

Narendra and Balakrishnan [12] describe the intelligent control as the ability of a controller to operate in multiple environments by recognizing which environment is currently in existence and servicing it appropriately. Conventional robust control is restricted to sufficiently small ranges of variations, and conventional adaptive control reacts slowly to abrupt changes. Multiple model switching adaptive control can respond to sudden and large changes immediately.

The usage of a multiple model approach requires to drive the process operating regime into many smaller local operating regimes and to associate them with a simple model structure, generally linear. In a number of applications of modeling with switching, the self-organizing map has been utilized to divide the operating region into local regions [13]. The SOM based on competitive learning implements an orderly mapping of a high-dimensional distribution to a regular low-dimensional grid [14]. It is able to convert complex, nonlinear statistical relationships between high-dimensional data items into simple geometric relationships on a low-dimensional display, and preserve the topology in the feature space. In the process of developing a generic on-line learning control system based on neural dynamic programming, Si and Wang [15] placed the SOM prior to an action network. They used the SOM as a state classifier to compress state measurements into a smaller set of vectors represented by the weights of the network, in order to reduce the learning complexity in the action network.

Inverse control of plant dynamics involves feed forward compensation, driving the plant with a filter whose transfer function is the inverse of the plant [16]. Wang et al. [17] utilized an inverse feed forward controller for conducting polymer actuator displacement control. Feed forward plant inversion has a number of issues which must be addressed such as causality, high gain, robustness, and unstable inverse dynamics. Unstable zeros are common in system models which have noncollocated sensors and actuators. Gross and Tomizuka [18] proposed the truncated series approach to deal with the unstable zeros in inverse control.

In this paper, we focus on system identification and vibration control of a pneumatic driven piezoelectric flexible manipulator. Parametric estimation methods are utilized in system identification of the piezoelectric manipulator subsystem. For the pneumatic driven system, a single global linear model is not accurate to describe its dynamics, due to the high nonlinearity of the pneumatic driven system. Therefore, a SOM based multi-model system identification approach is used to obtain the multiple local linear models. A SOM based multi-model inverse controller and a variable damping pole-placement controller are investigated in accordance with the different characteristics of pneumatic drive and piezoelectric actuator.

The rest of this article is organized as follows. Section 2 begins with a brief introduction of the experimental system. Section 3 concentrates on the system identification of the pneumatic driven piezoelectric flexible manipulator dynamics. A SOM based multi-model inverse controller and a variable damping pole-placement controller are developed in Section 4. Section 5 presents the experimental results. Finally, conclusions of this work are drawn in Section 6.

2. Description of the experimental setup

2.1. Brief device description

The laboratory setup serves as a test stand to verify the investigated strategies. The schematic diagram of a pneumatic driven piezoelectric flexible manipulator is shown in Fig. 1. The flexible manipulator is made of epoxide resins with one end clamped and fixed to a rigid slider and the other allowed vibrating freely. The beam dimensions are 650 x 100 x 1.78 mm.
The density of the epoxide resin material is 1840 kg/m³, Poisson’s ratio 0.33, and its Young’s modulus is 34.64 GPa. The aim is to drive the flexible manipulator to a desired position while minimizing the vibration of the flexible manipulator at the same time. According to the previous researches, the actuators must be placed at locations to excite the desired modes most effectively. Since the first two bending modes are the desired modes, the location close to the fixed end of the beam is the optimal location. Five PZT patches are glued on the beam close to the fixed end. One of the PZT patches is used as sensor to generate signal proportional to the mechanical deformation, while the other four patches are glued symmetrically on both sides of the beam, used as a one-channel PZT actuator. The geometric size of the PZT patches is 50 mm in length, 15 mm in width and 1 mm in thickness. The mechanical parameters of the PZT patches are as follows: Young’s elastic modulus 63.0 GPa, Poisson’s ratio 0.3, and density 7650 kg/m³, respectively. There is an accelerometer mounted at the tip of the flexible beam. In this research, the accelerometer is not used as a sensor, but it is used as a concentrated mass to reduce the frequency of the beam.

The actuator is a double-acting single rod pneumatic cylinder mounted on a base. The slider is rigidly mounted to the rod and moved only in the horizontal direction. The flexible manipulator can reach a targeted location within the range of the cylinder’s stroke. The displacement of the slider is detected by a linear potentiometer fixed on the base, and its sliding part is connected with the slider by using bolts via a universal joint. The stroke of the pneumatic rod cylinder is 150 mm. Its bore diameter and rod diameter are 32 mm and 12 mm, respectively. The measurement range of the linear potentiometer is 300 mm, with output voltage signals ranging from 0 to 10 V.

The pressure of the air source $P_s$ is firstly regulated by a pneumatic triplet. Then, three relief valves $P_{s1}$, $P_{s2}$ and $P_{s0}$ are used to regulate the pressures of the left and right chambers of the pneumatic cylinder and the back pressure circuit. The pressures of three relief valves are set as $P_{s1} = 0.4$ MPa, $P_{s2} = 0.5$ MPa and $P_{s0} = 0.3$ MPa. A check valve is connected with the relief valve $P_{s0}$ in the back pressure circuit and the 5-2 way switching valve $V_1$. The switching direction of the cylinder is controlled by a 5-2 way fast switching solenoid valve $V_1$. A proportional valve $V_2$ is used in the pneumatic servo system and connected with the 5-2 way switching valve $V_1$ and the exit of the check valve. Because of introducing the check valve, the pressure gas in the exhausted chamber must flow through the proportional valve $V_2$ which adjusts the orifice area of the exhaust pressure. A throttle valve is connected to the proportional valve in cascade mode. When the piston moves left, the supply pressure is $P_{s2}$ and the exhaust pressure is regulated by the proportional valve $V_2$. When the piston moves right, the supply pressure is $P_{s1}$. The pressure difference between the two chambers moves the load back and forth.

The picture of the experimental device is shown in Fig. 2. The experimental system includes supplementary instruments such as a charge amplifier for PZT sensor, voltage amplifier for PZT actuator and data acquisition system. In the experimental system, the air compressor (SLG50, Botelli Machinery) can provide an exhaust pressure of 0.8 MPa. The double-acting single rod pneumatic cylinder (SMC, CM2RB32-150) is horizontally installed on the base. Three relief valves (AR2500) are used to adjust the pressures of both chambers of the cylinder and the back pressure circuit, respectively. The fast 5-2 way switching valve (VK3120, SMC Corporation) can control the motion direction of the piston. The switching valve is driven through a valve driving board, which receives TTL signals from the GPIO port of the ARM board. The proportional valve (SMC, ITV2050-212L) adjusts the orifice area of the exhaust pressure.

2.2 Functional scheme of the device

Fig. 3 illustrates the functional scheme of the laboratory device. The system is a dual-input dual-output system. Controllers with a graphical user interface are developed in C++ language and run on a PC. A PC communicates with an ARM board (Mini2440) via RS-232 serial port. The ARM board is used as a data acquisition and control board. The data acquisition

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**Fig. 1.** Schematic diagram of the pneumatic driven flexible manipulator.

**Fig. 3.** The functional scheme of the laboratory device.
The ARM board provides analog-to-digital (A/D) and digital-to-analog (D/A) conversion with 12-bit resolution, which is made by ourselves. The slider displacement and the vibration of the flexible manipulator are measured by the potentiometer and the PZT sensor, respectively. The potentiometer provides analog outputs to the A/D converter with voltage between 0 V and 10 V. The voltage of the vibration signal is between \(-10 \text{ V}\) and \(10 \text{ V}\), amplified by a charge amplifier before sending to the A/D converter. Two D/A converters are used to drive the piezoelectric actuators through a voltage amplifier and the proportional valve. The voltage amplifier accepts an input signal between \(-5 \text{ V}\) and \(5 \text{ V}\) and then amplifies it up to \(\pm 130 \text{ V}\) peak. The proportional valve accepts analog input with voltage between 0 V and 5 V. To drive the slider back and forth, the proportional valve must cooperate with a switching valve, which determines the direction of the velocity of the slider. The ARM board turns on or turns off the switching valve when necessary.

3. System identification

3.1. Subsystem description

The pneumatic driven piezoelectric flexible manipulator is a dual-input dual-output system. Fig. 4 is the block diagram. It can be divided into two subsystems: the pneumatic driven subsystem and the piezoelectric driven subsystem. System identifications are implemented for each subsystem, separately.
In Fig. 4, $r_1$ is the reference signal of the slider position control. The desired displacement is specified $r_1 = 75 \text{ mm}$. A PD controller is used for position control. $u_1$ is the proportional valve control value. It is a linear superposition of the outputs of PD control and the pneumatic driven vibration controller. $y_1$ is the displacement of the slider and $y_2$ is the vibration signal of the flexible beam measured by the PZT sensor.

The parameters of the PD controller are determined by using several experimental tests. The mathematical model is used in the pneumatic driven vibration controller design. The mathematical model can be divided into two parts, i.e., the first part from $y_1$ to $y_2$ obtained from system identification; the second part from $u_1$ to $y_1$ which is not enough to describe by a single linear model due to the nonlinear dynamics of the pneumatic driven system. Therefore, a SOM based multiple model approach is used in system identification.

Fig. 5 shows the block diagram of the PZT driven flexible manipulator vibration control system. The reference signal $r_2$ for vibration control is equal to zero. The PZT actuator control value is $u_2$. $y_2$ is the measured vibration signal.

### 3.2. System identification procedure

In general, system identification consists of five basic steps: experiment design, data acquisition, selection of the model structure, parameter estimation, and model validation [19]. Prior system knowledge, modeling objectives, and observed data are the main components of the system identification procedure, where prior knowledge plays a key role. Different schemes of system identification for the different subsystems are selected based on the properties of each subsystem.

The autoregressive with external input (ARX) model structure takes the form [20]

$$A(q)y(k) = B(q)u(k),$$  \hspace{1cm} (1)

with

$$A(q) = 1 + a_1 q^{-1} + \ldots + a_{n_a} q^{-n_a},$$  \hspace{1cm} (2)

and

$$B(q) = b_1 q^{-1} + \ldots + b_{n_b} q^{-n_b},$$  \hspace{1cm} (3)

where $q$ denotes the forward-shift operator, $[qy(k) = y(k+1)]$. The inverse of the forward-shift operator $q^{-1}$ is called the backward-shift operator or the delay operator, $[q^{-1}y(k) = y(k-1)]$. $n_a$ and $n_b$ are the model orders.

The linear regression employs a predictor

$$\hat{y}(k|\theta) = \varphi^T(k)\theta,$$  \hspace{1cm} (4)

where $\varphi$ is the regression vector and $\theta$ is the parameter vector.

For the ARX model structure, the regression vector is

$$\varphi(k) = \begin{bmatrix} -y(k-1) & -y(k-2) & \ldots & -y(k-n_a) & u(k-1) & \ldots & u(k-n_b) \end{bmatrix}^T,$$  \hspace{1cm} (5)

and the parameter vector is

$$\theta = \begin{bmatrix} a_1 & a_2 & \ldots & a_{n_a} & b_1 & \ldots & b_{n_b} \end{bmatrix}^T.$$  \hspace{1cm} (6)

Introducing the notations

$$Y(k) = \begin{bmatrix} y(1) & y(2) & \ldots & y(k) \end{bmatrix}^T,$$  \hspace{1cm} (7)

and

$$\Phi(k) = \begin{bmatrix} \varphi^T(1) \\ \vdots \\ \varphi^T(k) \end{bmatrix},$$  \hspace{1cm} (8)

and assuming, that the matrix $\Phi^T\Phi$ is nonsingular, the solution to the least squares problem is unique and given by

$$\theta = (\Phi^T\Phi)^{-1}\Phi^TY.$$  \hspace{1cm} (9)

Middleton and Goodwin [21] proposed a different discrete time method for discretizing of continuous time models. The fundamental difficulty with a forward-shift operator or z-transform is avoided by use of an alternative operator, namely the
Delta operator $\delta$. The relationship between Delta operator $\delta$ and forward-shift operator $q$ is
\begin{equation}
\delta = q - \frac{1}{T_s}.
\end{equation}

As the sampling time goes to zero, the $\delta$ operator tends to the differential operator. The models obtained through the $\delta$ operator converge to their original continuous models as the sampling time approaches zero, which is not the case in commonly used discrete time models. The sensitivity of pole location process leads to small changes in the identified parameters is much less in the $\delta$ domain than it is in the $z$ domain.

For a continuous time transfer function
\begin{equation}
G(s) = \frac{B(s)}{A(s)}
\end{equation}

When the relative degree $r$ between $G(s) = \frac{B(s)}{A(s)}$ and $B(s)$ meets the relationship $r > 1$, an approximate discrete time model can be obtained such that the local truncation error between the output of this model and the true system is of order $O(T_s^{r-1})$ [22]. Replacing Laplace operator $s$ by $\delta$, and taking the relative degree into consideration, the approximate discrete time model is given by
\begin{equation}
G_r(\delta) = \frac{B(\delta)P_r(T_s,\delta)}{A(\delta)},
\end{equation}

where
\begin{equation}
P_r(T_s,\delta) = \text{det}
\begin{bmatrix}
1 & \frac{T_s}{\delta} & \cdots & \frac{T_s^{r-1}}{\delta} \\
-\delta & 1 & \cdots & \frac{T_s^{r-1}}{\delta} \\
\vdots & \ddots & \ddots & \ddots \\
0 & \cdots & -\delta & 1
\end{bmatrix}.
\end{equation}

In Eq. (13), when the relative degree $r = 2$, $P_r(T_s,\delta) = \frac{T_s}{\delta} + 1$. This will be used in system identification.

The main difficulty in handling continuous time models is the presence of the derivative operator associated with the input and output signals. Discretization in the Delta domain, as an alternative to the conventional shift operator domain, avoids undue sensitivity at high sampling rates. The Delta operator is backward differences based approximation of the derivative operator and it results in accentuation of noise. Filtering of the experimental data is necessary before parameters are estimated in the procedure of system identification.

3.3. System identification of the PZT drive subsystem

Piezoelectric beam can be described by a linear model as long as the control value of the PZT actuators is not over saturated. The experimental data used in system identification is the vibration response excited by the PZT actuator with swept sine signal. The swept sine frequency range is specified from 1 Hz up to 20 Hz in 50 s.

Fig. 6 shows the experimental results of vibration response excited by PZT actuator with a swept sine signal. The first two bending modes of vibration are shown in Fig. 6(c). The natural frequencies of the first mode and the second mode are $f_1 = 2.7$ Hz and $f_2 = 17.1$ Hz, respectively.

Considering the first bending mode, the model structure obtained by the parameter estimation method is
\begin{equation}
H_1(z) = \frac{b_{11}z^{-1} + b_{12}z^{-2}}{1 + a_{11}z^{-1} + a_{12}z^{-2}}.
\end{equation}

Then, the regression vector in Eq. (4) is $q(k) = \begin{bmatrix} -y_2(k-1) & -y_2(k-2) & u_2(k-1) & u_2(k-2) \end{bmatrix}^T$ and the parameter vector is $\theta_1 = [a_{11} \ a_{12} \ b_{11} \ b_{12}]^T$.

As previously noted, the swept sine signal applied to the PZT actuator includes the first and the second transversal resonant mode. The data set used for system identification of the first bending mode should exclude the data of the second mode of vibration. The selected data set is ranging from 1 s to 25 s. By using the least squares estimation in Eq. (9), the parameter vector $\theta_1$ is obtained as $\theta_1 = [1.9887 \ 0.9957 \ 2.5859 \times 10^{-5} \ -1.6315 \times 10^{-5}]^T$. So the discrete time transfer function of the first bending mode is
\begin{equation}
H_1(z) = \frac{2.586 \times 10^{-5} z^{-1} - 1.632 \times 10^{-5} z^{-2}}{1 - 1.989 z^{-1} + 0.996 z^{-2}}.
\end{equation}

Model validation is a crucial step in the modeling process. The validation data come from the experiments of the PZT drive active vibration control of the flexible beam by using PD control. The method of model validation is $n$-step ahead model prediction. Let $u_2(k)$ denote the voltage applied to the PZT actuator at the sampling instance $k$; let $y_2(k)$ denote the vibration signal measured by the PZT sensor, and let $\hat{y}_2(k)$ denote the predicted vibration signal. $\hat{y}_2(k)$ is computed from past data
\begin{equation}
u_2(k-1), \ldots, u_2(k-n-1), y_2(k-n), y_2(k-n-1), \ldots
\end{equation}

using the model (15).
The case when \( n \) equals 1 corresponds to the one step ahead prediction. The one step ahead prediction is not a good indicator of the model accuracy for this system. Due to the fast sampling rate, the discrepancy between one step ahead predicted vibration signal \( \hat{y}_2(k) \) and the measured vibration signal \( y_2(k) \) will be very small even for an inaccurate model. The horizon length \( n \) should be increased to validate the model accuracy.

Consider the process model of only the second bending mode. The model structure used by the parameter estimation method is same as that of the first bending mode. The data set used for system identification of the second bending mode ranges from 45 s to 50 s. By using the least squares estimation in Eq. (9), the discrete time transfer function of the second bending mode is obtained as

\[
H_2(z) = \frac{1.297 \times 10^{-4}z + 3.986 \times 10^{-5}}{z^2 - 1.718z + 0.996}.
\] (17)

The natural frequency of the second bending mode obtained by the parameter estimation method is 16.988 Hz. Fig. 6 (b) shows that the natural frequency of the second bending mode is 17.1 Hz. Thus, the difference is not significant. The natural frequency obtained by the parameter estimation method depends on the selected data set. When the data set used for system identification is ranging from 40 s to 50 s, the natural frequency obtained is 16.83 Hz. The PZT drive vibration controller is designed based on the transfer function (17).

3.4. System identification of the pneumatic driven subsystem

Since the pneumatic drive subsystem is open loop unstable (and for the safety of the flexible beam), it is necessary to perform the identification experiment under closed loop. If there is feedback from the output to the input in some regulators
the spectral and correlation analysis estimates are not reliable. A system can be defined by the state variable model

\[ \begin{align*}
    \dot{x} &= Ax + Bu \\
    y &=Cx
\end{align*} \tag{18} \]

and the control feedback given by

\[ u = kx. \tag{19} \]

Then, the closed-loop system is

\[ \begin{align*}
    \dot{x} &= (A + BK)x \\
    y &=Cx
\end{align*} \tag{20} \]

The closed-loop system can also be written as

\[ \begin{align*}
    \dot{x} &= (A + (1 - \alpha)BK)x + \alpha Bu \\
    y &=Cx
\end{align*} \tag{21} \]

where \( \alpha \) is an arbitrary constant.

When the state matrix \( A \) is known as prior system knowledge, \( \alpha \) is equal to one. There is a unique estimated control matrix \( B \) which can give the best input–output description of the system. From the velocity signal of the slider to the vibration signal of the flexible beam, the transfer function is

\[ G_{xy}(s) = \frac{B(s)}{A(s)} \tag{22} \]

Since the limitation of the control bandwidth of the pneumatic driven system, the vibration frequency beyond the first mode cannot be controlled. The transfer function \( G_{xy}(s) \) is limited to a second order system. \( A(s) \) represents the properties of the flexible beam; it is equal to the denominator of the transfer function of the PZT drive subsystem, that is \( A(s) = s^2 + 2\xi\omega_1 s + \omega_1^2 \). Let \( B(s) = b_1 s + b_0 \), where \( b_1 \) and \( b_0 \) are the parameters needing to be identified. By substituting \( A(s) \) and \( B(s) \) into Eq. \( (22) \), the transfer function becomes

\[ G_{xy}(s) = \frac{b_1 s + b_0}{s^2 + 2\xi\omega_1 s + \omega_1^2} \tag{23} \]

Since \( b_1 \) and \( b_0 \) are unknown, the relative degree \( r \) between \( A(s) \) and \( B(s) \) is also unknown. For system identification, the relative degree \( r \) is assumed to be equal to one firstly. If \( b_1 \) can be ignored compared to \( b_0 \), the parameters need to be identified again with \( r \) equal to two. Assuming \( r = 1 \) and according to Eq. \( (12) \) the approximate discrete time model in \( \delta \) domain is given by

\[ G_{xy}(\delta) = \frac{b_1 \delta + b_0}{\delta^2 + 2\xi\omega_1 \delta + \omega_1^2} \tag{24} \]

Assuming \( r = 2 \), the approximate discrete time model in \( \delta \) domain is given by

\[ G_{xy}(\delta) = \frac{b_0(4T_{\delta}\delta + 1)}{\delta^2 + 2\xi\omega_1 \delta + \omega_1^2} \tag{25} \]

Let consider the case that the relative degree \( r \) is equal to one. Then, the discrete time model \( (24) \) can be transformed to a linear regression model as

\[ z(k) = \Phi^T(k)\theta, \tag{26} \]

where \( z(k) = (\delta^2 + 2\xi\omega_1 \delta + \omega_1^2)y(k), \Phi(k) = [\delta u(k) \quad u(k)^T ]^T \) and \( \theta = [b_1 \quad b_0]^T \).

By using the least squares method, the parameter vector \( \theta \) can be obtained. A linear potentiometer provides the displacement of the slider \( y \). Undesired noise and disturbances are always present in the laboratory and will affect measurements in various ways. The velocity of the slider \( v \) is not measured directly and any attempt to measure it either by numerical computation or signal processing through a differentiator will result in amplification of noise. A curve fitting method is used to obtain the velocity \( v \) from the position data set which leads no phase shift.

The parameter equation used to fit the position data set is a series of a third-degree polynomial with the form

\[ y(x) = ax^3 + bx^2 + cx + d. \]

The velocity \( v(k) \) is calculated from \( y(k-m), y(k-m+1), \ldots, y(k), \ldots, y(k+m) \). Here, \( m \) is a selected constant. Using data shifting, \( g(-m) = y(k-m), g(-m+1) = y(k-m+1), \ldots, g(0) = y(k), \ldots, g(m-1) = y(k+m-1), g(m) = y(k+m) \) the algorithm of fitting the polynomial curve can be realized by the data pairs \( (n, g(n)) \). Here, \( n = -m, -m+1, \ldots, m; g(n) \) is the shifting data. After determining the parameters of the polynomial, the velocity is

\[ v(k) = \frac{\sum_{i=0}^{m} g(i)}{m}. \]

Fig. 7 shows the experimental results used in system identification. The data are obtained from composite position and vibration control by the PD controller. The reliability of these data is verified by the subsequent SOM based multi-model vibration control experiments. In Fig. 7(c), significant high frequency noise is visible in the velocity signal. From Fig. 7(d) and
(e), it can be known that the velocity curve becomes more and more smooth with the increasing of $m$. $m$ should not be too large to avoid the loss of more details in the velocity curve. The velocity data set obtained with $m = 15$ is used for system identification.

Fig. 7. Experimental results used in system identification. (a) Control voltage applied to the proportional valve. (b) Displacement curve of the slider. (c) Velocity obtained by using numerical differentiation. (d) Velocity obtained by curve fitting with $m = 10$. (e) Velocity obtained by curve fitting with $m = 15$. (f) Vibration signal of the flexible beam.
The subsystem model from the velocity signal of the slider to the vibration signal of the flexible beam is obtained by applying the least squares method on the data set shown in Fig. 7(e) and (f). The parameter vector $\theta$ in Eq. (26) is obtained as $\theta = \begin{bmatrix} 0.1316 \\ -0.2300 \end{bmatrix}$, thus, $b_1 = 0.1316$ and $b_0 = -0.2300$. By substituting $b_1$ and $b_0$ into Eq. (23), the transfer function becomes

$$G_{vf}(s) = \frac{0.1316s - 0.2300}{s^2 + 0.3054s + 287.7977}.$$  \hspace{1cm} (27)

Since $b_1$ cannot be ignored compared to $b_0$, the assumption that the relative degree $r$ is equal to one is valid.

SOM is based on competitive learning. The output neurons of the network compete among themselves, with the result that only one output neuron is on at any one time. Fig. 8 illustrates a SOM. In the SOM, the neurons are placed on the nodes of a two-dimensional lattice. An output neuron which wins the competition is called a winner-takes-all neuron or simply a winning neuron. The neurons become selectively tuned to various input patterns during the competitive learning process. A SOM is characterized by the formation of a topographic map of the input patterns. The coordinates of the neurons in the lattice are an expression of intrinsic statistical features contained in the input patterns, hence the name “self-organizing map.”

The algorithm responsible for the formation of the SOM proceeds first by initializing the synaptic weights in the network. This can be done by assigning to them small values picked from a random number generator. Once the network has been properly initialized, there are three essential processes involved in the formation of the SOM, as summarized here:

1. Competition. For each input pattern, the neurons in the network compute their respective values of a discriminant function. This discriminant function provides the basis for competition among the neurons. The particular neuron with the largest value of discriminant function is declared winner of the competition.

2. Cooperation. The winning neuron determines the spatial location of a topological neighborhood of excited neurons, thereby providing the basis for cooperation among such neighboring neurons.

3. Synaptic adaptation. This last mechanism enables the excited neurons to increase their individual values of the discriminant function in relation to the input pattern through suitable adjustments applied to their synaptic weights. The adjustments are made in such a way that the response of the winning neuron to the subsequent application of a similar input pattern is enhanced.

Let $m$ denote the dimension of the input space. Let an input pattern be denoted by

$$\mathbf{x} = [x_1 \ x_2 \ \ldots \ x_m]^T.$$  \hspace{1cm} (28)

The synaptic weight vector of each neuron in the network has the same dimension as the input space. Let the synaptic weight vector of neuron $j$ be denoted by

$$\mathbf{w}_j = [w_{j1} \ w_{j2} \ \ldots \ w_{jm}]^T, \ j = 1, 2, \ldots, l$$  \hspace{1cm} (29)

where $l$ is the total number of neurons in the network.

The discriminant function is chosen as the inner product $\mathbf{w}_j^T \mathbf{x}$. The best matching criterion based on maximizing the inner product $\mathbf{w}_j^T \mathbf{x}$ is mathematically equivalent to minimizing the Euclidean distance between the vectors $\mathbf{x}$ and $\mathbf{w}_j$. The Euclidean distance between the vector $\mathbf{x}$ and $\mathbf{w}_j$ is

$$f(\mathbf{x}) = ||\mathbf{x} - \mathbf{w}_j||.$$  \hspace{1cm} (30)

Let assume, that the index $i(\mathbf{x})$ identifies the winning neuron of the input vector $\mathbf{x}$.

$$i(\mathbf{x}) = \arg\min_j f(\mathbf{x}), \ \ j = 1, 2, \ldots, l.$$  \hspace{1cm} (31)
Depending on the application of interest, the response of the network could be the index of the winning neuron, the synaptic weight vector of the winning neuron, or additional information associated with the winning neuron. For the SOM based multi-model inverse control, the additional information associated with the winning neuron is the parameters of a local linear model.

The winning neuron locates the center of a topological neighborhood of cooperating neurons. The variable \( h_{ji} \) denotes the lateral distance between winning neuron \( i \) and excited neuron \( j \), and it is

\[
d^2_{ji} = \| r_j - r_i \|^2.
\]

where the vector \( r_j \) defines the position of excited neuron \( j \) and \( r_i \) defines the position of winning neuron \( i \) in the two-dimensional lattice.

The variable \( h_{ji} \) denotes the topological neighborhood centered on winning neuron \( i \) and encompassing an excited neuron \( j \). A typical choice of \( h_{ji} \) is a Gaussian function as

\[
h_{ji} = \exp \left( -\frac{d_{ji}^2}{2\sigma^2} \right).
\]

where \( \sigma \) represents the effective width of the topological neighborhood.

A popular choice of \( \sigma \) on discrete time \( n \) is the exponential decay described by

\[
\sigma(n) = \sigma_0 \exp \left( -\frac{n}{\tau_1} \right), \quad n = 0, 1, 2, \ldots,
\]

where \( \sigma_0 \) is the value of \( \sigma \) at the initiation of the SOM algorithm, and \( \tau_1 \) is a time constant.

As time \( n \) increases, the width \( \sigma(n) \) decreases at an exponential rate, and the neighborhood function \( h_{ji} \) shrinks in a corresponding manner.

To create the network for SOM, the synaptic weight vector \( w_j \) of neuron \( j \) in the network is required to change in relation to the input vector \( x \). Given the synaptic weight vector \( w_j(n) \) of neuron \( j \) at time \( n \), the updated weight vector \( w_j(n+1) \) at time \( n+1 \) is defined as

\[
w_j(n+1) = w_j(n) + \eta(n) h_{ji}(n) (x - w_j(n)),
\]

where \( \eta(n) \) is the learning rate parameter.

The learning rate parameter should be decreased gradually with increasing time \( n \), written as

\[
\eta(n) = \eta_0 \exp \left( -\frac{n}{\tau_2} \right).
\]

where \( \tau_2 \) is another time constant of the SOM algorithm.

The SOM is utilized to divide the operating regions into local regions. The SOM modeling is done in the output space. The regression vector in Eq. (5) can be rewritten as

\[
\varphi(k) = \begin{bmatrix} -\varphi_y(k) \\ \varphi_u(k) \end{bmatrix}.
\]

where \( \varphi_y(k) = \begin{bmatrix} y(k-1) \cdots y(k-n_a) \end{bmatrix} \) and \( \varphi_u(k) = \begin{bmatrix} u(k-1) \cdots u(k-n_b) \end{bmatrix} \).

The experimental data in the form of \( \varphi_y(k) \) is applied to train the SOM. To divide the operating region, the experimental data are applied to the SOM again. However, the synaptic weight vector is not updated. One should find the winning neuron \( i \) corresponding to the input vector \( \varphi_y(k) \) and the corresponding vector \( \varphi_u(k) \) of the winning neuron \( i \). Then, the \( i \)th local linear model can be obtained by using parameter estimation.

The mathematical model of the pneumatic driven system has been established in [23]. If the control signal is held constant and the cylinder was capable of infinite movement, then the velocity of the slider would tend to a constant value. This suggests that there is an integrator in the transfer function from the control voltage of the proportional valve \( u \) to the displacement of the slider \( y_1 \). Due to the compressibility of air, the transient velocity response of the slider \( v \) to a step input control voltage \( u \) is a damped oscillation. So the transfer function from the control voltage \( u \) to the displacement of the slider \( y_1 \) can be expressed as

\[
G_{uy_1}(s) = \frac{b_1 s + b_0}{s(s^2 + a_1 s + a_0)}
\]

where \( a_0, a_1, \) and \( b_1 \) are the parameters to be estimated.

And then the transfer function from the control voltage \( u \) to the velocity of the slider \( v \) is

\[
G_{uv}(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0}
\]
Assuming that \( b_1 \) cannot be ignored, the relative degree of transfer function (39) is \( r = 1 \). Then the approximate discrete time model of transfer function (39) in \( \delta \) domain is given by

\[
G_{uv}(\delta) = \frac{b_1 \delta + b_0}{\delta^2 + a_1 \delta + a_0}.
\]  

(40)

The parameters in Eq. (40) are estimated by applying the least squares method on the data set. The transfer function is obtained as

\[
G_{uv}(s) = \frac{36.89s + 7033}{s^2 + 13.19s + 359.3}.
\]  

(41)

Since \( b_1 = 36.89 \) is much smaller than \( b_0 = 7033 \), \( b_1 \) can be ignored and the relative degree is \( r = 2 \). The approximate discrete time model of transfer function (39) in \( \delta \) domain is given by

\[
G_{uv}(\delta) = \frac{b_0 (\delta T_s + 1)}{\delta^2 + a_1 \delta + a_0}.
\]  

(42)

Applying the parameter estimation by using the least squares method on the data set shows in Fig. 7(a) and (e), the transfer function is obtained as

\[
G_{uv}(s) = \frac{7097}{s^2 + 13.26s + 362.8}.
\]  

(43)

SOM based multi-model system identification is performed to obtain the subsystem model from the control voltage of the pneumatic proportional valve to the velocity of the slider. The input vector of SOM can be selected as the time series of displacement or velocity of the slider. The SOM is utilized to divide the operating region into local regions. Previous researchers have found that for pneumatic actuators, the dynamic characteristic is piston position dependent \([24,25]\). The displacement or velocity of the slider. The SOM is utilized to divide the operating region into local regions. Previous researches have found that for pneumatic actuators, the dynamic characteristic is piston position dependent \([24,25]\). The input vector of SOM in our experiments is some different as compared with the Eq.(37) for the convenience of programming. The input vector has the form of \([y_{1}(k-3)\ y_{1}(k-2)\ y_{1}(k-1)]\). The reorder of the time series data reflected in the synaptic weight vector automatically. It has no influence on the division of operating region. In the SOM based multi-model control strategy, the number of local models is determined by the dimension of SOM lattice. A large number of local models lead to the smooth curve of controller parameters, and then result in smooth control signal. However, the computational load in real-time control will increase as the increase of local models. The dimension of SOM lattice should be selected according to how dramatically the dynamical system characteristics vary over the operating regime and the computing power. The data used for identification is 800 set. The SOM is selected as a \( 5 \times 5 \) lattice. Thus, there is 800/25 = 32 set data for each SOM lattice. This will meet the requirement for experiments. The dimension of the synaptic weight vectors is the same as the input vector which is 3.

Since the input vector is selected as the time series of the displacement, the synaptic weight vectors are initialized according to the displacement curve of the slider illustrated in Fig. 7(b). The desired displacement of the slider is \( r_1 = 75 \) mm in experiments. Considering the overshoot of the position control, the displacement of the slider is between 0 and 80 mm. So the synaptic weight vectors are initialized with random numbers distributed uniformly between 0 and 80.

\[
\theta = \frac{1}{\sqrt{N}} \begin{bmatrix} u(k-3) & u(k-2) & u(k-1) \end{bmatrix}.
\]

After that, the data set of displacement in the form of \([y_{1}(k-3)\ y_{1}(k-2)\ y_{1}(k-1)]\) is applied to training the SOM. The weight vectors of the SOM after training are illustrated in Table 1.

The found winning neuron corresponds to the input vector \( \theta = \frac{1}{\sqrt{N}} \begin{bmatrix} u(k-3) & u(k-2) & u(k-1) \end{bmatrix} \). \( r_{\text{index}} \) and \( c_{\text{index}} \) represent the number of rows and the number of columns, respectively. The neurons distributed in the two-dimensional lattice are indexed as

\[
j = 5 \cdot (r_{\text{index}} - 1) + c_{\text{index}}.
\]  

(44)

The index of winning neuron versus time is illustrated in Fig. 9.

The data set of a neuron includes the time series of displacement \( \begin{bmatrix} y_{1}(k-3) & y_{1}(k-2) & y_{1}(k-1) \end{bmatrix} \), the corresponding time series of velocity \( \begin{bmatrix} v(k-3) & v(k-2) & v(k-1) \end{bmatrix} \) and control voltage \( \begin{bmatrix} u(k-3) & u(k-2) \end{bmatrix} \). A local linear model can be obtained by applying the least squares method on the data set of time series of velocity and control voltage of a neuron. However, the data set of one neuron cannot establish a local linear model, since the number of data pairs is less than the modeling requirement, or the control voltages are zero for all data pairs. Thus, we made use of data samples from the winner as well as the neighbors to create the local models. Data samples from different neurons are weighted to create the local linear models by the weighted least squares method.

Multiplying both sides of Eq. (1) by a weight factor \( \alpha(k) \) leads to

\[
\alpha(k)y(k|\theta) = \alpha(k)\Theta^T(k|\theta).
\]  

(45)
Table 1
Weight vectors of the SOM.

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</table>

Fig. 9. Index of winning neurons versus time.

The parameter vector $\theta$ should be chosen to minimize the least squares lost function

$$V(\theta, n) = \frac{1}{2\Theta} \sum_{k=1}^{n} \alpha^{2}(k)(y(k) - \phi^{T}(k)\theta)^{2}. \quad (46)$$

This is called the weighted least squares method. By replacing the original $y(k)$ by $\alpha(k)y(k)$, and replacing the original $\phi^{T}(k)$ by $\alpha(k)\phi^{T}(k)$, the Eq. (9) can also be used to get the solution to the weighted least squares problem.

To obtain the local linear model for neuron $i$, the data samples of neuron $i$ are weighted by

$$\alpha_{ji} = \frac{1}{||r_{j} - r_{i}|| + \beta}$$

(47)

where the vector $r_{j}$ and $r_{i}$ defines the position of the neuron $j$ and $i$ in the two-dimensional lattice, respectively; $||r_{j} - r_{i}||$ represents the distance between the neuron $j$ and $i$; $\beta$ is a positive constant in case $j$ is equal to $i$.

As the distance between neuron $j$ and $i$ increases, the weight factor $\alpha_{ji}$ decreases. The rate of decrease is controlled by $\beta$. The data samples of the neuron $j$ have the largest weight factor $1/\beta$. The constant $\beta$ is selected to be equal to one. For each neuron, a local linear model is obtained by the weighted least squares method. The parameters of twenty five local linear models corresponding to each neuron are listed in Table 2. For example the local linear model corresponding to the slider at mid-stroke belongs to the neuron at the fifth row and the first column, as listed in Table 2. In such a case, the transfer
where x is the input matrix; Γ is the input matrix; u is the input signal; y is the output signal; L is the gain matrix of the Kalman filter and is to be determined as part of the filter design procedure.

The Kalman filter can be simplified by excluding the input signal. It could be done due to the reason that the Kalman filter can be thought of as operating in two distinct phases: predict and update. The predict phase uses the state estimate from the previous step to produce an estimate of the state at the current step. In the update phase, the prediction is combined with current observation to refine the state estimate. The input signal is used in the predict phase. Without the input signal, prediction can still be performed with a lower accuracy. The prediction error can be compensated by the update phase. Since the sampling frequency is very high, the contribution of the input signal to the predicted state is far smaller than the contribution of the self-iteration of the state vector. Therefore, the input signal could be ignored. In such a case, the
simplified Kalman filter is given by
\[ \hat{x}(k) = \Phi \hat{x}(k-1) + L(y(k) - C \hat{x}(k-1)). \] (51)

The input signal to the Kalman filter is the vibration signal \( y_2 \) measured by the PZT sensor. The estimated state vector \( \hat{x} \) is a four-dimensional vector written as \( \hat{x} = [\hat{x}_1 \ \hat{x}_2 \ \hat{x}_3 \ \hat{x}_4]^{T} \); where \( \hat{x}_1 \) and \( \hat{x}_2 \) denote the vibration signal of the first mode and its differential signal, respectively; \( \hat{x}_3 \) and \( \hat{x}_4 \) represent the vibration signal of the second mode and its differential signal, respectively. The output matrix is \( C = [c_1 \ 0 \ c_2 \ 0] \). The discrete-time state matrix \( \Phi \) with zero-order hold sampling is expressed as [26]
\[ \Phi = I + A \Psi, \]
\[ \Psi = \int_{0}^{T_s} e^{As} ds = IT_s + \frac{A^2 T_s^2}{2} + \frac{A^3 T_s^3}{3!} + \cdots + \frac{A^{i-1} T_s^{i-1}}{(i-1)!} + \cdots, \] (52)
where \( T_s \) is the sampling interval and \( A \) is the continuous-time state matrix written as
\[ A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_1^2 & -2\xi_1 \omega_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_2^2 & -2\xi_2 \omega_2 \end{bmatrix}, \] (53)
where \( \omega_1 \) and \( \omega_2 \) are the natural frequencies of the first mode and the second mode, respectively; \( \xi_1 \) and \( \xi_2 \) are the damping ratios of the first mode and the second mode, respectively.

---

Fig. 10. Model parameters versus time.
In order to check the properties of the Kalman filter, the output is defined as
\[ \hat{y}(k) = H\hat{x}(k). \] (54)

If \( H \) is chosen in a proper way, the outputs of the first and the second vibration modes can be separated. \( H = \begin{bmatrix} c_1 & 0 & 0 & 0 \end{bmatrix} \) and \( H = \begin{bmatrix} 0 & 0 & c_2 & 0 \end{bmatrix} \) are the Kalman filter’s outputs of the first mode and the second mode, respectively. Fig. 11 shows the Bode diagram of the designed Kalman filter.

From Fig. 11(a), it can be known that the amplitude-frequency response is almost horizontal around the natural frequency of the first mode and has a deep trough around the frequency of the second vibration mode. Furthermore, this does not cause phase lag at the natural frequency of the first vibration mode. In the case of the Kalman filter with the second mode of vibration as output (Fig. 11(b)), the amplitude will not be attenuated at the natural frequency of the second mode, while a greatly attenuation happens at the natural frequency of the first vibration mode. Thus, the signal decoupling of the first and the second mode of vibration can be accomplished by using the designed Kalman filter. The controllers of the first and the second modes can be designed accordingly.

Fig. 11. Bode diagram of the designed Kalman filter. (a) Bode diagram of the Kalman filter with the first mode of vibration as output. (b) Bode diagram of the Kalman filter with the second mode of vibration as the output.
4.2. Pneumatic driven piezoelectric flexible manipulator under PD control

There are two purposes of conducting PD control experiments. The first aim is that the experimental results under PD control are used for system identification and model validation. The second one is that the experimental results under PD control are compared with those under the designed SOM based multi-model inverse control and the variable damping pole-placement control, to show the performance of the investigated control algorithms.

Two PD controllers are utilized: one for position control of the slider of the pneumatic driven subsystem, and the other for vibration control of the flexible manipulator. The composite PD control is given by

\[
 u_1 = K_p (r_1 - y_1) - K_d y_1 - K_p y_2 - K_d y_2, \tag{55}
\]

where \( r_1 \) is the reference signal of the slider position control; \( y_1 \) and \( y_2 \) are the displacement of the slider and the vibration signal of the flexible beam measured by the PZT sensor, respectively; \( K_p \) and \( K_d \) are the proportional and derivative gains of the PD controller for position control, respectively.

The control voltage of the PD controller applied to the PZT actuator is

\[
 u_2 = -K_p y_2 - K_d y_2, \tag{56}
\]

where \( K_p \) and \( K_d \) are the proportional and derivative gains of the PZT drive PD controller, respectively.

4.3. SOM based multi-model inverse control and pole placement controller design

A SOM based multi-model inverse controller is designed. As previously mentioned, the mathematical model of the pneumatic driven subsystem can be divided into two parts. The first mathematical model is from the proportional valve control value \( u_1 \) to the velocity of the slider \( v \). The second is from the velocity of the slider \( v \) to the vibration signal \( y_2 \). A linear feedback controller is designed for the second part of the pneumatic driven subsystem. The phase angle of the transfer function (23) which described dynamics from \( v \) to \( y_2 \) is about 6° at the natural frequency of the first bending mode of the flexible beam. Thus, a proportional negative feedback controller can suppress the vibration effectively. Let \( K \) denote the proportional gain. The closed-loop transfer function from the slider’s velocity \( v \) to the vibration signal \( y_2 \) with proportional negative feedback control is

\[
 H_{vy} (s) = \frac{0.1316}{s^2 + (0.3054 + 0.1316K)s + 287.7977 - 0.2300K} \tag{57}
\]

The damping ratio of the closed-loop transfer function is increased by the proportional negative feedback controller. The slider’s velocity is regulated by the proportional valve, which is controlled by an analog voltage signal. Thus, the proportional negative feedback controller should be extended to deal with the dynamics from the proportional valve control value \( u_1 \) to the velocity of the slider \( v \).

Fig. 12 shows the SOM based multi-model inverse controller. A SOM based inverse model is placed between the output of the proportional negative feedback controller \( v_c \) and the control voltage applied to the proportional valve \( u_1 \). The dynamics from \( u_1 \) to \( v \) is compensated by the inverse model. It is expected that the slider’s velocity \( v \) will be approximately equal to the output of the proportional negative feedback controller \( v_c \) because of introducing the inverse model.

The SOM based multiple inverse models are obtained directly from the SOM based multi-model system identification results. It is composed of multiple local inverse models, and each one is the inverse of a local linear model. Because these local linear models can be expressed by transfer functions in the form of Eq. (39) with \( b_1 = 0 \), they have the inverse models

Fig. 12. Block diagram of the SOM based multi-model inverse controller.
in the form
\[ C_{vu_1} = \frac{s^2 + a_1 s + a_0}{b_0}. \] (58)

During online control, the SOM based multi-model method is used to select the appropriate local inverse model. At time \( k \), the input to the SOM is \([y_1(k-2) \ y_1(k-1) \ y_1(k)]\). After completing the competitive process, the local inverse model belongs to the winning neuron is selected for inverse control. For each sampling interval, the controller consists of a local inverse model and a proportional negative feedback. The transfer function of the controller has the following form
\[ C = K \frac{s^2 + a_1 s + a_0}{b_0}, \] (59)
and the proportion valve control value is
\[ u_1 = \frac{-K(\dot{y}_2 + a_1 \dot{y}_2 + a_0 y_2)}{b_0}. \] (60)

Due to the bandwidth limitation of the pneumatic drive used in experiments, only the vibration of the first bending mode can be effectively suppressed by the pneumatic cylinder. To conduct experiments, the state \( y_2 \) and \( y_2 \) in Eq. (60) are replaced by the estimated state \( \hat{x}_1 \) and \( \hat{x}_2 \) obtained from the Kalman filter. Since the vibration signal \( y_2 \) is approximately a sine wave in some sense, \( \hat{y}_2 \) can be approximated by \( -\omega_1^2 y_2 \). Therefore, the control voltage applied to the proportional valve can be expressed as
\[ u_1 = \frac{-K \left[ (-\omega_1^2 + a_0) \hat{x}_1 + a_1 \hat{x}_2 \right]}{b_0}. \] (61)

The acting force of the PZT patch is comparatively small. To suppress the vibration of flexible manipulator quickly, the control gain should be specified high. Then, the control voltage applied to the PZT actuator will be saturated for a long time. In order to avoid the continuous saturation of the PZT actuator control voltage, a variable damping pole-placement controller is designed.

For the discrete-time process model expressed by Eq. (14), the control value is given by
\[ u(k) = \frac{1}{b_1} [y^*(k+1) - \hat{y}(k+1) - b_2 u(k-1)], \] (62)
where
\[ y^*(k+1) = -a_1^T y(k) - a_2^T y(k-1), \] (63)
and
\[ \dot{y}(k+1) = -a_1 y(k) - a_2 y(k-1). \] (64)
where \( y^*(k+1) \) represents the desired output at the next sampling instance; \( \hat{y}(k+1) \) is the predicted output using the information from the previous step; \( a_1^T \) and \( a_2^T \) are the corresponding parameters.

By choosing \( a_1^T \) and \( a_2^T \), the autonomous system \( y^*(k+1) + a_1^T y^*(k) + a_2^T y^*(k-1) = 0 \) will have desired closed-loop poles. Then, the desired characteristics of the control system can be achieved.

The control strategy expressed in Eq. (62) is a special form of pole-placement control. Fig. 13 shows the two degrees of freedom R-S-T digital controller. The digital filters \( R \) and \( S \) are designed to achieve the desired regulation performance, and the digital filter \( T \) is designed afterwards to achieve the desired tracking performance. In Fig. 13, \( B(z)/A(z) \) is the transfer function model of the control process. Since vibration control is a regulation problem, one can select \( T(z) = 1 \). The control strategy (62) is obtained by letting \( A(z) = 1 + a_1 z^{-1} + a_2 z^{-2}, B(z) = b_1 z^{-1} + b_2 z^{-2}, A^*(z) = 1 + a_1^* z^{-1} + a_2^* z^{-2}, R(z) = A^*(z) - A(z), S(z) = B(z), T(z) = 1 \) and \( r = 0 \). The closed-loop transfer function from \( r \) to \( y \) is
\[ H(z) = \frac{1}{A^*(z)}. \] (65)

The control strategy (62) is modified to deal with the first two bending mode of vibration. The output matrix \( C \) in Eq. (51) can be partitioned into two blocks \( C = [C_1 \ C_2] \), where \( C_1 = [c_1 \ 0] \) and \( C_2 = [c_2 \ 0] \). The estimate of the state vector \( \hat{x} \) is partitioned into \( \hat{x} = [\hat{x}_1^T \ \hat{x}_2^T]^T \) accordingly, where \( \hat{x}_1 = [x_1 \ x_2]^T \) and \( \hat{x}_2 = [x_3 \ x_4]^T \). The state matrix \( \Phi \) is partitioned into \( 2 \times 2 \) blocks
\[ \Phi = \begin{bmatrix} \Phi_1 & 0 \\ 0 & \Phi_2 \end{bmatrix}, \] (66)
where \( \Phi_1 \) and \( \Phi_2 \) represent the state matrix of the first and the second bending mode, respectively.
Let $\Phi_1^*$ and $\Phi_2^*$ denote the desired state matrix of the first and the second bending mode, respectively. The control value applied to the PZT actuator is

$$u(k) = \left[ \begin{array}{c} \frac{1}{b_{11}} \\
\frac{1}{b_{22}} \end{array} \right] \left[ \begin{array}{c} C_1(\Phi_1^* - \Phi_1)\dot{x}_1 - b_{12}u(k-1) \\
C_2(\Phi_2^* - \Phi_2)\dot{x}_2 - b_{22}u(k-1) \end{array} \right].$$

(67)

where $b_{11}$, $b_{12}$, $b_{21}$ and $b_{22}$ are the parameters in transfer functions (15) and (17).

The discrete-time desired state matrix $\Phi_1^*$ and $\Phi_2^*$ are calculated according to Eq. (52). The corresponding continuous-state matrixes are

$$A_1 = \begin{bmatrix} 0 & 1 \\
-\omega_1^2 & -2\xi_1^* \omega_1 \end{bmatrix}.$$  

(68)

Fig. 13. Pole placement with the R-S-T controller.

Fig. 14. Position control of the pneumatic slider with PD control. (a) Displacement curve of the slider. (b) Vibration signal of the flexible beam. (c) Control voltage of the proportional valve.
and

\[
A_2^* = \begin{bmatrix}
0 & 1 \\
-\omega_2^2 & -2\xi_2^* \omega_2
\end{bmatrix},
\]

where \(\xi_1^*\) and \(\xi_2^*\) are the expected damping ratios need to be selected for the first and the second mode.

If the expected damping ratios are selected too large, the control value applied to the PZT actuator will be saturated for a long time. On the other hand, the vibration cannot be suppressed quickly with too small damping ratios. The natural frequency of the second bending mode \(\omega_2\) is fairly large compared to the natural frequency of the first bending mode \(\omega_1\). Furthermore, the vibration of the second mode decays relatively quickly. Thus, the expected damping ratio of the second mode is selected as a fixed appropriate value. The expected damping ratio of the first mode \(\xi_1^*\) is adjusted dynamically, depending on the control value and the saturation voltage of the voltage amplifier. It is given by

\[
\left\{ \begin{array}{l}
\xi_1^*(k) = \xi_0 e^{-h \lambda(k)} + \xi_1,

\lambda(k) = \frac{\sum_{i=1}^{n} |u(k-i)|}{n \cdot u_{\text{max}}},
\end{array} \right.
\]

where \(\xi_0\) is a constant value; \(\xi_1\) is the damping ratio of the first mode without closed-loop control; \(h\) is a scaling factor; the saturation value of the voltage amplifier is \(\pm u_{\text{max}}\); \(\xi_1\) is added in case that \(\xi_1^*\) is smaller than \(\xi_1\).

\[\text{(70)}\]

\[\text{Fig. 15. Simultaneously position and vibration control with using pneumatic cylinder under PD control. (a) Displacement curve of the slider. (b) Vibration signal of the flexible beam. (c) Control voltage applied to the pneumatic proportional valve.}\]
When the mean value of $\frac{1}{n} \sum_{i=1}^{n} |u(k-i)|$ achieves to $u_{\text{max}}$, the expected damping ratio $\xi_{1}^{*}$ goes to $\xi_0 e^{-h} + \xi_1$. This phenomenon can be avoided if the scaling factor $h$ is large enough. When the mean value of control value $\frac{1}{H} \sum_{i=1}^{n} |u(k-i)|$ achieves to zero, the expected damping ratio $\xi_{1}^{*}$ goes to $\xi_0 + \xi_1$. Small amplitude vibration can be suppressed fast with a large $\xi_{1}^{*}$.

5. Experimental results

The experiments include: position control of the pneumatic slider only, composite position and vibration control only using pneumatic cylinder, vibration suppression only by using the PZT actuator and hybrid position and vibration control using both the pneumatic cylinder and the PZT actuator. Three kinds of control strategies are applied, namely, PD control, the SOM based multi-model inverse control and the variable damping pole-placement controller. In the subsequent experiments, the desired displacement of the slider is specified as 75 mm, and the control effect is applied to drive the pneumatic cylinder at the moment of $t = 0.5$ s. The sensitivity of the charge amplifier is set as 358 pC/Unit, and its output relationship is 0.1 mV/Unit. At the beginning of experimental tests, the vibration of the flexible beam is excited, and the measured vibration amplitude is limited in the range from $-10$ V to $+10$ V.

5.1. Case study 1: position control only

For position control of the slider using PD control, the parameters of the PD controller are chosen as $K_{p1} = 8 \times 10^{-2}$ and $K_{d1} = 4 \times 10^{-3}$.

Fig. 16. Pneumatic driven vibration control by using SOM based inverse control. (a) Displacement curve of the slider. (b) Vibration signal of the flexible beam. (c) Control voltage applied to the proportional valve. (d) Index of a winning neuron.
From Fig. 14(a), one can know that accurate position control is achieved by the designed PD controller. Fig. 14(b) reveals that the low frequency large amplitude vibration will last for a long time without active vibration control. Fig. 14(c) shows that the control voltage reaches the maximum value. The control capability for positioning is sufficient.

5.2. Case study 2: composite position and vibration control with using only pneumatic drive

For simultaneous pneumatic driven position and vibration control, two PD controllers should be designed. The controller parameters are chosen as $K_{p1} = 6 \times 10^{-2}$, $K_{d1} = 4 \times 10^{-3}$, $K_{p2} = 2 \times 10^{-2}$, $K_{d2} = 2 \times 10^{-3}$. The control voltage applied to the pneumatic proportional valve is the linear superposition of the outputs of the two PD controllers. Therefore, the parameters of the PD controller are reduced compared to the case of merely position control.

Fig. 15 shows experimental results of simultaneous position and vibration control. From Fig. 15, one knows that the low frequency large amplitude vibration will last for a long time without active vibration control. Fig. 14(c) shows that the control capability for positioning is sufficient.

5.2. Case study 2: composite position and vibration control with using only pneumatic drive

For simultaneous pneumatic driven position and vibration control, two PD controllers should be designed. The controller parameters are chosen as $K_{p1} = 6 \times 10^{-2}$, $K_{d1} = 4 \times 10^{-3}$, $K_{p2} = 2 \times 10^{-2}$, $K_{d2} = 2 \times 10^{-3}$. The control voltage applied to the pneumatic proportional valve is the linear superposition of the outputs of the two PD controllers. Therefore, the parameters of the PD controller are reduced compared to the case of merely position control.

Fig. 15 shows experimental results of simultaneous position and vibration control. From Fig. 15, one knows that the low frequency large amplitude vibration will last for a long time without active vibration control. Fig. 14(c) shows that the control capability for positioning is sufficient.

To suppress the vibration further, the SOM based multi-model inverse controller is studied next. The PD controller for pneumatic driven vibration control is replaced by the SOM based multi-model inverse controller, and the parameters of the
PD controller for position control are not changed. The proportional gain $K$ in the SOM based multi-model inverse controller is specified as 3.

Fig. 16 presents the experimental results of simultaneously position and vibration control under PD control and SOM based multi-model inverse control, respectively. Comparing Fig. 16(b) with Fig. 15(b), one can see that the vibration amplitude of the flexible manipulator is suppressed better with the SOM based multi-model inverse controller. The index of winning neuron in Fig. 16(d) indicates the switching between multiple inverse models.

Comparing Fig. 16(c) with Fig. 15(c), it can be known that the maximum control values are almost equal. Because of introducing the SOM based multi-model inverse controller, the vibration is attenuated to smaller amplitude.

5.3. Case study 3: vibration suppression using only the PZT actuator

The amplified gain of the high voltage amplifier is 26. The parameters of the designed PD controller are selected as $K_{p3} = 0.8$ and $K_{d3} = 0.08$. Fig. 18 shows the experimental results of vibration control under PD control. Fig. 17(a) shows that the vibration is attenuated by the PZT actuator comparing with the free vibration in Fig. 14(b). Since the acting force of the PZT actuator is relatively weak for the small amplitude vibration, the vibration is still visible at time $t = 10$ s. Vibration can be suppressed faster by increasing the gains of the PD controller. Fig. 18 shows the experimental results by a high-gain PD controller with parameters as $K_{p3} = 1.2$, $K_{d3} = 0.15$. The vibration is damped out after $t = 7$ s. However, the control voltage is saturated for about 3.5 s, as presented in Fig. 18(b).

The parameters of the variable damping pole-placement controller are specified as $\xi_0 = 0.1$, $n = 30$, $u_{\text{max}} = 130$ and $h = 2$. Fig. 19(a) depicts the measured vibration signal of the flexible beam. Fig. 19(b) shows the control voltage applied to the PZT actuator; Fig. 19(c) illustrates the adjusting process of the expected damping ratio $\xi_1^*$. Comparing Fig. 19(a) with Fig. 17(a), it
can be seen that the vibration is suppressed much better using variable damping pole-placement control. The saturation time of control voltage in Fig. 19(b) is shorter than in Fig. 17(b) or Fig. 18(b). Fig. 19(c) shows the value of damping ratio increasing along with the decreasing of control voltage and the vibration signal. The saturation time of control value is reduced due to the small damping ratio. Vibration of the flexible beam is suppressed faster with a large damping ratio. This is because the variable damping control law is applied. The damping of the closed-loop system is increasing with the decrease of the vibration amplitude. The improved vibration control performance validates the efficiency of the variable damping pole-placement controller for the PZT actuator.

5.4. Case study 4: hybrid position and vibration control using both the pneumatic and PZT actuators

The pneumatic control can suppress the large amplitude vibration of the first bending mode effectively. Small amplitude vibration including higher mode can be suppressed by the PZT actuator. The advantages of both the actuators are combined together to achieve a better active vibration control performance.

Experiments on hybrid position and vibration control using both the actuators by PD control are conducted. The control gains of three PD controllers are specified as $K_{p1} = 6 \times 10^{-2}$, $K_{d1} = 4 \times 10^{-3}$, $K_{p2} = 3 \times 10^{-2}$, $K_{d2} = 4 \times 10^{-4}$, $K_{p3} = 0.6$, $K_{d3} = 0.05$. The experimental results are shown in Fig. 20. Comparing Fig. 20(b) with Fig. 15(b), it can be known that the large amplitude vibration of the first mode is attenuated mainly by the pneumatic driven system. After that, the PZT actuator can suppress the small amplitude vibration including the higher mode. The introduced PZT actuator can damp out the vibration finally. However, the vibration control time is much longer when the designed PD controller is utilized.

Fig. 20. Hybrid position and vibration PD control using both the pneumatic cylinder and the PZT actuator. (a) Displacement curve of the slider. (b) Vibration signal of the flexible beam. (c) Control voltage applied to the proportional valve. (d) Control voltage applied to the PZT actuator.
In the second experiment, the SOM based multi-model inverse controller and the variable damping pole-placement controller (SOM-VDP) have been utilized for the pneumatic driven system and the PZT actuator in vibration control, respectively. A PD controller was implemented.

Fig. 21. Hybrid pneumatic drive and PZT actuator SOM-VDP vibration control for the first mode. (a) Displacement curve of the slider. (b) Vibration signal of the flexible beam. (c) Control voltage applied to the proportional valve. (d) Control voltage applied to the PZT actuator. (e) Index of a winning neuron. (f) Adjusting process of the damping ratio.
Firstly, only the first bending mode of vibration is considered. Parameters of the PD controller for position control are selected as \( K_{p1} = \frac{6}{C2} \times 10^{-2} \) and \( K_{d1} = 4.8 \times 10^{-3} \). The proportional gain \( K \) in the SOM based multi-model inverse controller is equal to 3. The parameters of the variable damping pole-placement controller are specified as \( \xi_0 = 0.1 \), \( n = 30 \), \( u_{\text{max}} = 130 \) and \( h = 2 \).

![Graphs showing displacement, vibration signal, control voltage, and adjusting process of damping ratio](image)

**Fig. 22.** Hybrid pneumatic drive and PZT actuator SOM-VDP vibration control for the first two modes. (a) Displacement curve of the slider. (b) Vibration signal of the flexible beam. (c) Control voltage applied to the proportional valve. (d) Control voltage applied to the PZT actuator. (e) Index of a winning neuron. (f) Adjusting process of the damping ratio.
Fig. 21 shows the experimental results of the hybrid pneumatic drive and PZT actuator SOM-VDP vibration control for the first mode. The results show that the SOM based multi-model inverse controller and the variable damping pole-placement controller lead to a significantly improved damping of the first bending mode. From Fig. 21(b), it can be seen that the second mode is excited by the pneumatic drive and will last for a long time without effective control. This is because that only the vibration of the first bending mode is controlled by the PZT actuator by applying the designed controller.

In the next step, the vibration of the first two bending modes was considered. The vibration of the second mode is suppressed by the PZT actuator with pole-placement controller. The expected damping ratio for the second mode is $\xi_2^* = 0.02$. For a second order linear model with damping ratio equal to 0.02 and natural frequency equal to 17.1 Hz, the amplitude of vibration can attenuate to its $1/10$ in one second.

Fig. 22 shows the experimental results of the hybrid pneumatic drive and PZT actuator SOM-VDP vibration control for the first two modes. Comparing Fig. 22(b) with Fig. 21(b), one knows that better vibration control performance is achieved by combining the advantages of both the pneumatic and PZT actuators for the first two bending modes of vibration. Both the large and the small amplitude vibration can be damped out quickly using the investigated control algorithm. The proposed hybrid drive vibration control strategy is feasible to simultaneous position and vibration control.

6. Conclusions

A kind of hybrid pneumatic driven piezoelectric flexible manipulator system, for simultaneous position and vibration control of a flexible piezoelectric manipulator has been presented. The system is composed of a pneumatic proportional valve for pneumatic driven system and a piezoelectric control system of the flexible beam. After establishing the experimental setup, system identification methods are employed to obtain the system dynamics models. Parameter estimation methods are applied for the piezoelectric flexible manipulator; and the SOM based multiple linear models are obtained, for the pneumatic driven system. Experiments are carried out in view of different situations using the classical PD controller; a SOM based multi-model inverse controller and a variable damping pole-placement controller. The experimental results demonstrate that the hybrid pneumatic and piezoelectric control scheme can suppress the vibration of the flexible manipulator effectively. Furthermore, the investigated SOM based multi-model inverse control algorithms can improve the control performance of the investigated system, as compared with the classical PD controller.

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