

# Varying Inertial Parameters Model Based Robust Control for An Aerial Manipulator\*

been funded in Europe, and many aerial manipulators have

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**Abstract**—The aerial manipulator is a new kind of UAV system composed of an aerial robot and a multi-link manipulator. It enables the traditional aerial robots to perform fine manipulation, and thus has great potential for many real applications. However, one of the most challenging problems of the aerial manipulator system is the high performance steady control. Its difficulties mainly come from the heavy dynamics couplings between the aerial robot and the manipulator, especially when quick relative motion is required. Thus in this paper, the modeling and motion control problem of the aerial manipulator is studied. Firstly, a new modeling scheme is proposed by using the concept of Varying Inertial Parameters (VIP), the main idea of which is to take the influence of coupling on the aerial robot as a group of state dependent inertial parameters, i.e., the Center of Mass (CoM) and inertia matrix of the aerial robot is varying with the relative state variance. Subsequently, based on the VIP model, a feedforward compensation  $H_\infty$  controller is designed to implement the steady flight of the whole system with relative movement. Finally, simulations are conducted and the feasibility and validity of the new proposed scheme is shown.

## I. INTRODUCTION

In the past ten years, rotorcraft UAV has attracted great attention from researching community and thus has achieved great development. Recently, a new emergent topic on the study of rotorcraft UAV is the aerial manipulator system. Usually, an aerial manipulator is composed of a rotorcraft UAV and an operating mechanism, such as gripper or manipulator. Because of wide application prospect, the researches on the aerial manipulator has attracted great attentions and some work on this topic has also been published. For example, in [1], a miniature quadrotor equipped with a 3-DoF (Degree of Freedom) delta structure arm is designed to accomplish contact measurement tasks. In [2], a scissor is equipped on a quadrotor for canopy sampling. In [3], a dual-arm aerial manipulator is designed to turn valve in some disaster scenarios. In [4], a gripper is attached to the bottom of a quadrotor to construct cubic structures with magnetic joints. Moreover, two projects on this topic has

been built<sup>[5][6]</sup>.

One of the great challenges for an aerial manipulator system to complete fine manipulation tasks is its safe and steady control. As shown in [7]-[10], dynamics model of the aerial manipulator system is extremely complicated due to the coupling effects between the rotorcraft UAV and the manipulator, especially when quick relative motion is required. So, it is difficult to analyze system dynamics behavior and simplify the model.

Recently, some researches have shown that the influence of motion of manipulator on the rotorcraft UAV can be taken as changement of the UAV's mass distribution. In [11] and [12], the authors conduct extensive experiments, and the results show that the changement of the manipulator significantly influence the motion steadiness of the UAV. They finally point out that this kind of influence can be eliminated by restricting the movement of the manipulator in the lateral plane of the UAV. In [3], increment of torque due to the manipulator's joint rotating motion are taken into account in system dynamics model to stabilize the aerial robot. In [13], offsets of the aerial manipulator's CoM due to grasped payload is considered when modeling translational dynamic model, and then in [14], they give out the stability bound within which the changing mass-inertia parameters of the aerial manipulator system will not destabilize the rotorcraft UAV controlled by standard PID. In [15], the influence of mass distribution changement on aerial manipulator is analyzed and a variable gain that depends on the arm joint angle positions is used in variable parameter integral backstepping controller. Some researches also have been done to minimize the displacement of system CoM. For example, in [16], a 5-DoF light-weight robot arm is designed with a differential joint and motors are relocated at the base of the arm to remain the CoM close to the body frame. In [17], a mechanism is proposed considering a moving battery to counterweight the statics of the robotic arm to decrease displacement of CoM of aerial manipulator system.

Although the above works consider the influence of the manipulator's movement as the changement of the inertial parameters, most of them try to find some ways to minimize this influence through manipulator trajectory planning or designing new mechanism, and none of them focuses on the detailed mathematical model to describe and analyze the problem. This makes it hard to design proper and effective controller to realize the steady flight of the whole system. Thus, in this paper, a new modeling scheme, called Varying Inertial Parameters (VIP) model, to quantitatively consider variation of the mass distribution is proposed. The main idea is to introduce a group of variable inertia parameters to

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describe influence of manipulator on the UAV. Based on the VIP model, the dynamics behavior of aerial manipulator system is analyzed, and a feedforward compensation linear  $H_\infty$  flight controller is designed to eliminate the unsteadiness due to the relative motion between the UAV and the manipulator. The rest of this paper is organized as follows. In section II, the VIP model is introduced and the dynamics behavior of the aerial manipulator is analyzed and explained based on this new model; Then, in section III the new flight controller, which is called feedforward compensation linear  $H_\infty$  control is designed based on the VIP model. Subsequently, validity and feasibility of the new proposed controller is verified by simulations and the results are given and analyzed in section IV. Finally, conclusions and future work are listed out in section V.

## II. VARIABLE INERTIA PARAMETERS MODEL

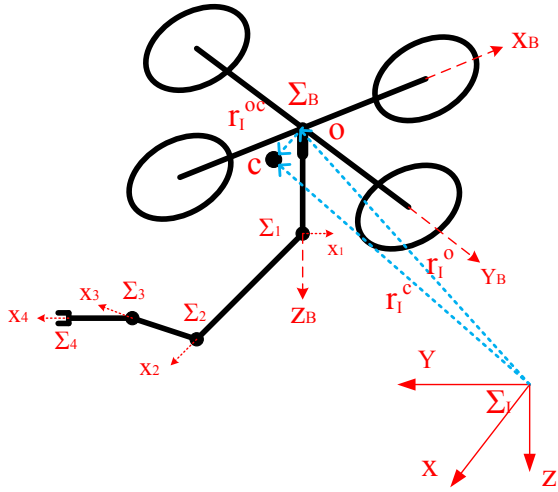


Figure.1 coordinate frame of aerial manipulator

### A. Definition of varying inertia parameters

In this paper, we consider an aerial manipulator composed by a quadrotor and a four DOF manipulator. The system and the coordinates are given as in Fig. 1. In Fig. 1,  $\Sigma_I$  denote the NED inertial coordinate frame and  $\Sigma_B$  is the body fixed coordinate frame, where  $X_B$ ,  $Y_B$ , and  $Z_B$  are axes of  $\Sigma_B$ , and the coordinate origin  $O$  coincides with the CoM of quadrotor. Joint coordinate frames are constructed based on standard DH parameters [18] and denoted by  $\Sigma_i$  ( $i=1,2,3,4$ ) for each joint.

When the manipulator is moving, the inertial parameters (CoM and inertia matrix) of the aerial manipulator system changes accordingly, which can be calculated based on robot kinematics as follows<sup>[18]</sup>.

$$r_B^{OC}(q) = \frac{\sum_{i=1}^4 m_i r_B^{OCi}(q)}{m_Q + \sum_{i=1}^4 m_i} \quad (1)$$

where,

- $q$ : joint angle vector of the manipulator.
- $r_B^{OCi}(q)$ : position of CoM of link  $i$  with respect to  $\Sigma_B$ . It is a function of  $q$ .

- $r_B^{OC}(q)$ : position of CoM of the aerial manipulator with respect to  $\Sigma_B$ .
- $m_Q$ : mass of the quadrotor.
- $m_i$ : mass of the link  $i$  of the manipulator.

Based on the definition of inertia matrix and parallel axis theorem, the inertia matrix of the aerial manipulator can be obtained as following

$$I_B^C(q) = I_B^O + \Delta I_B^C(q) \quad (2)$$

$$\Delta I_B^C(q) = -m_Q \cdot skew^2(r_B^{OC}(q)) + \sum_{i=1}^4 (R_i^B I_i^C (R_i^B)^T - m_i \cdot skew^2(r_B^{OCi}(q) - r_B^{OC}(q))) \quad (3)$$

where,

- $I_B^C(q)$ : inertia matrix of the aerial manipulator referenced to the system's CoM along to body fixed frame axes with respect to  $\Sigma_B$ .
- $I_B^O$ : inertia matrix of quadrotor referenced to point  $O$  along to frame axes of  $\Sigma_B$  with respect to  $\Sigma_B$ .
- $\Delta I_B^C(q)$ : variation of inertia matrix referenced to the system CoM along to body fixed frame axes with respect to  $\Sigma_B$ .
- $R_i^B$  is rotation matrix from  $\Sigma_i$  to  $\Sigma_B$ .
- $I_i^C$  is inertia matrix of link  $i$  referenced to CoM of link  $i$  along to frame axes of  $\Sigma_i$  with respect to  $\Sigma_i$ .
- $skew(.)$  is skew symmetric matrix function of a vector.

### B. Dynamics model of aerial manipulator

In this section, The origin of  $\Sigma_B$  is denoted by  $\zeta=(x,y,z)^T$  in  $\Sigma_I$ , and the quadrotor attitude is described by the  $Z$ - $Y$ - $X$  Euler angles  $\eta=(\phi,\theta,\psi)^T$ , denoting roll, pitch and yaw angle, respectively.  $R$  denotes rotation matrix from  $\Sigma_B$  to  $\Sigma_I$ , and expressed as follows

$$R = \begin{bmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\ c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix}$$

where,  $c$  and  $s$  denote trigonometric function  $\cos(.)$  and  $\sin(.)$ , respectively. Define unit vectors  $e_1, e_2, e_3$ :

$$[e_1 \quad e_2 \quad e_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

#### 1) Translational dynamic

As shown in Fig.1, position of point  $O$  and the system's CoM are denoted by vector  $r_i^O$  and  $r_i^C$ , respectively, and they are related to each other by the following equation,

$$r_i^C = r_i^O + R r_B^{OC}(q) \quad (4)$$

So, the acceleration of the system's COM can be easily obtained as follows,

$$a_i^C = \ddot{r}_i^C = \ddot{r}_i^O + R(\dot{\omega}_B \times r_B^{OC}(q) + \omega_B \times (\omega_B \times r_B^{OC}(q))) + R(2\omega_B \times \dot{r}_B^{OC}(q) + \ddot{r}_B^{OC}(q)) \quad (5)$$

where  $a_i^C$  is acceleration of the CoM with respect to  $\Sigma_i$ ;  $\omega_B$  is the angular velocity of the quadrotor with respect to

$\Sigma_B$ . From Eq. (1)  $r_B^{OC}(q)$  is independent on the quadrotor's state. That means it can be seen as external inputs. Define  $\delta_1 = [r_B^{OC}(q); \dot{r}_B^{OC}(q); \ddot{r}_B^{OC}(q)]$ , so  $a_I^C$  can be rewritten as,

$$a_I^C = \ddot{r}_I^O - d_1(\eta, \omega_B, \dot{\omega}_B, \delta_1) \quad (6)$$

where

$$d_1(\eta, \omega_B, \dot{\omega}_B, \delta_1) = -R[skew(\dot{\omega}_B) + skew^2(\omega_B), 2skew(\omega_B), 1]\delta_1 \quad (7)$$

Based on Newton's equations of motion, the translational dynamic equation can be written as follows,

$$\dot{\xi} = v \quad (8)$$

$$\dot{v} = ge_3 + \frac{1}{m_s} FRe_3 + d_1(\eta, \omega_B, \dot{\omega}_B, \delta_1) \quad (9)$$

where  $v = \dot{r}_I^O$  is the velocity of point  $O$ ;  $m_s = m_\rho + \sum_{i=1}^{i=4} m_i$  is total mass of the aerial manipulator system;  $F$  is total thrust of quadrotor generated by all rotating rotors with respect to  $\Sigma_B$ ;  $g$  is gravity acceleration.

## 2) Rotational dynamic

Considering the variation of inertia matrix, the rotational dynamic of the aerial manipulator is as follows,

$$\dot{\eta} = T(\eta)\omega_B \quad (10)$$

$$I_B^C(q)\dot{\omega}_B = M_B^C - \omega_B \times (I_B^C(q)\omega_B) \quad (11)$$

where

$$T(\eta) = \begin{bmatrix} 1 & s\phi \tan \theta & c\phi \tan \theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi \sec \theta & c\phi \sec \theta \end{bmatrix}$$

and  $M_B^C = (M_x, M_y, M_z)^T$  is total moments acting on CoM of the aerial manipulator with respect to  $\Sigma_B$ .

In order to separate external inputs  $q$  from system state variables, we define another external inputs vector  $\delta_2$  composed of the elements of upper triangle of matrix  $\Delta I_B^C(q)$ . So Eq. (11) can be rewritten as follows,

$$\dot{\omega}_B = -I_B^{O-1}(\omega_B \times (I_B^O \omega_B)) + I_B^{O-1} M_B^C + d_2(\omega_B, \delta_2) \quad (12)$$

where

$$d_2(\omega_B, \delta_2) = (I + I_B^{O-1} \cdot \Delta I_B^C(q))^{-1} \cdot I_B^{O-1} \cdot \Delta I_B^C(q) \cdot I_B^{O-1} \cdot (\omega_B \times (I_B^C(q)\omega_B)) - I_B^{O-1}(\omega_B \times (\Delta I_B^C(q)\omega_B)) \quad (13)$$

Based on the preceding equations, the translational and rotational dynamics of the aerial manipulator system can be summarized as follows,

$$\dot{\xi} = v \quad (14)$$

$$\dot{\eta} = T(\eta)\omega_B \quad (15)$$

$$\dot{v} = ge_3 + \frac{1}{m_s} FRe_3 + d_1(\eta, \omega_B, \dot{\omega}_B, \delta_1) \quad (16)$$

$$\dot{\omega}_B = -I_B^{O-1}(\omega_B \times (I_B^O \omega_B)) + I_B^{O-1} M_B^C + d_2(\omega_B, \delta_2) \quad (17)$$

The  $F$  and  $M_B^C$  produced by the rotors can be denoted as following equation[19],

$$\begin{bmatrix} F & M_x & M_y & M_z \end{bmatrix}^T = \begin{bmatrix} c_T & c_T & c_T & c_T \\ c_T r_{B,y}^{OC} & -c_T(d - r_{B,y}^{OC}) & c_T r_{B,y}^{OC} & c_T(d + r_{B,y}^{OC}) \\ c_T(d - r_{B,x}^{OC}) & -c_T r_{B,x}^{OC} & -c_T(d + r_{B,x}^{OC}) & -c_T r_{B,x}^{OC} \\ -c_Q & c_Q & -c_Q & c_Q \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \quad (18)$$

where  $C_T$  is thrust coefficient;  $C_Q$  is torque coefficient;  $d$  is the distance from center of rotor to the geometrical center of the quadrotor and  $\omega_i$  is angular velocity of rotor  $i$  ( $i=1,2,3,4$ ).

Eq. (14)-(18) are the so-called VIP model of the aerial manipulator.

Remark I: The aerial manipulator possesses much more nonlinearities than the quadrotor system. This can be explained from the following two aspects:

- On the one hand, the new system presents stronger coupling between translational and rotational dynamics. In Eq. (16), a new term  $d_1$  is introduced due to the addition of the manipulator. This will make its controller design more difficult.
- On the other hand, the relation between control inputs  $[F M_x M_y M_z]^T$  and the rotor speeds  $[\omega_1, \omega_2, \omega_3, \omega_4]^T$  of the quadrotor becomes more complicated due to the variation of the CoM. This can be clearly seen in Eq.(18), where  $M_x$  and  $M_y$  are effected by movement of CoM in Y and X direction of body frame axes.

Because of those new characteristics, the traditional inner-outer loop based flight controller [20], as well as the feedback linearization scheme may fail to ensure the stability and steady of the whole system, and some new control scheme need to be designed.

## III. CONTROLLER BASED ON VIP MODEL

It is usually difficult to design a good enough controller for the VIP model as Eq.(7) and Eq.(13). Generally, in order to complete manipulation tasks, aerial manipulator need to keep hovering near some objects. So it is possible to linearize the VIP model and design a linear robust controller. In this section a linear feedforward compensation H $\infty$  controller is proposed to attenuate disturbance introduced by the movement of the manipulator.

### A. Linearization

From Eq.(1)-Eq.(2), The inertia parameters can be calculated directly based on joint angle which is measurable. That is,  $d_1$  and  $d_2$  are measurable disturbances. We rewrite Eq.(16)-Eq.(17) as follows,

$$\dot{v} = ge_3 + \frac{1}{m_s} FRe_3 + \Delta F \quad (19)$$

$$\dot{\omega}_B = -I_B^{O-1}(\omega_B \times (I_B^O \omega_B)) + I_B^{O-1} M_B^C + \Delta M \quad (20)$$

where,

$$\Delta F = [\Delta F_1 \quad \Delta F_2 \quad \Delta F_3]^T = d_1(\eta, \omega_B, \dot{\omega}_B, \delta_1) \quad (21)$$

$$\Delta M = [\Delta M_1 \quad \Delta M_2 \quad \Delta M_3]^T = d_2(\omega_B, \delta_2) \quad (22)$$

Define  $X = [\xi; \eta; v; \omega_b]$  as state vector and  $u = [F, M_x, M_y, M_z]^T$  as input vector. The equilibrium point of the hovering modal is,

$$X_0 = [x_d, y_d, z_d, 0, 0, \psi_d, 0, 0, 0, 0, 0, 0]^T$$

and the steady-state input is  $u_0 = [m_s g, 0, 0, 0]^T$  [21]. Using the small perturbation theory, i.e.,  $X = X_0 + \Delta X$  and  $u = u_0 + \Delta u$ , the linearized equations of system (19)-(20) can be written as,

$$\Delta \dot{X} = A \Delta X + B \Delta u + [O_{6 \times 1}; \Delta F; \Delta M] \quad (23)$$

where

$$A = \begin{bmatrix} O_{3 \times 3} & O_{3 \times 3} & I_{3 \times 3} & O_{3 \times 3} \\ O_{3 \times 3} & O_{3 \times 3} & O_{3 \times 3} & I_{3 \times 3} \\ O_{3 \times 3} & A_{32} & O_{3 \times 3} & O_{3 \times 3} \\ O_{3 \times 3} & O_{3 \times 3} & O_{3 \times 3} & O_{3 \times 3} \end{bmatrix} A_{32} = \begin{bmatrix} -g \cdot s\psi_d & -g \cdot c\psi_d & 0 \\ g \cdot c\psi_d & -g \cdot s\psi_d & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} O_{8 \times 4} \\ B_{21} \end{bmatrix} B_{21} = \begin{bmatrix} -1/m_s & 0 & 0 & 0 \\ 0 & 1/I_{xx} & 0 & 0 \\ 0 & 0 & 1/I_{yy} & 0 \\ 0 & 0 & 0 & 1/I_{zz} \end{bmatrix}$$

Define  $Y_d = [0, 0, 0, 0]^T$ , the linearized model of the aerial manipulator can be written as,

$$\dot{X} = AX + BU + [O_{6 \times 1}; \Delta F; \Delta M] \quad (24)$$

$$Y = CX \quad (25)$$

where

$$U = \Delta u \quad \text{and} \quad C = \begin{bmatrix} I_{3 \times 3} & O_{3 \times 2} & 0 & O_{3 \times 6} \\ O_{3 \times 3} & O_{3 \times 2} & 1 & O_{3 \times 6} \end{bmatrix}.$$

### B. Feedforward compensation

In some cases, influence of the measured disturbances on the system can be wiped off directly using feedforward compensation in control inputs. However, it is hard to realize in underactuated system, due to absence of control input in some disturbed directions. In [22], input assignability is proposed to deal with this situation. Here we use this scheme to attenuate the disturbances due to movement of the manipulator. Firstly, define new variable  $\bar{X}$  as follows,

$$\bar{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 + \Delta F_2 / g \\ X_5 - \Delta F_1 / g \\ X_6 \\ X_7 \\ X_8 \\ X_9 \\ X_{10} + \Delta F_2 / g \\ X_{11} - \Delta F_1 / g \\ X_{12} \end{bmatrix}.$$

Combine it with Eq.(24)-Eq.(25), we can get

$$\dot{\bar{X}} = A\bar{X} + B(U + B^{-1}\Delta) \quad (26)$$

$$Y = C\bar{X} \quad (27)$$

where

$$\Delta = [O_{1 \times 8}, \Delta F_3, M_x + \Delta \ddot{F}_2 / g, M_y - \Delta \ddot{F}_1 / g, M_z]^T$$

$B^{-1}$  is pseudo-inverse matrix of  $B$  which means  $BB^{-1} = I_{12 \times 12}$  and  $B^{-1}B = I_{4 \times 4}$ . As Eq.(26)-Eq.(27),  $\Delta F_1$  and  $\Delta F_2$  are assigned into control inputs directions and can be eliminated directly. The term,  $B^{-1}\Delta$ , can be used as feedforward to wipe off the measured disturbance.

### $H_\infty$ controller

Based on the scheme in the last sub-section, the disturbances can be attenuated by feedforward compensation, but the controller requires the high order derivatives of  $\Delta F$ , which is unfortunately unknown. Even it can be obtained through estimator or observer, the estimated error, called residual errors, may still present serious influence on system performance. Thus in this section,  $H_\infty$  controller will be designed to attenuate these residual errors. Beforehand, we firstly rewritten the system (26)-(27) as follows,

$$\dot{\bar{X}} = A\bar{X} + B(U + B^{-1}\Delta) + De \quad (28)$$

$$Y = C\bar{X} \quad (29)$$

where

$$D = \begin{bmatrix} O_{3 \times 2} & O_{3 \times 2} \\ I_{2 \times 2} / g & O_{2 \times 2} \\ O_{4 \times 2} & O_{4 \times 2} \\ O_{2 \times 2} & I_{2 \times 2} / g \\ 0 & 0 \end{bmatrix} e = \begin{bmatrix} \Delta \dot{F}_2 - \Delta \ddot{F}_2 \\ -\Delta \dot{F}_1 + \Delta \ddot{F}_1 \\ \Delta \dot{F}_2 - \Delta \ddot{F}_2 \\ -\Delta \dot{F}_1 + \Delta \ddot{F}_1 \end{bmatrix}$$

$\Delta \dot{F}$  and  $\Delta \ddot{F}$  are estimation of  $\Delta \dot{F}$  and  $\Delta \ddot{F}$ , respectively. So we designed a linear  $H_\infty$  controller for system in Eq.(28)-Eq.(29) to ensure closed-loop stability. The objective of linear  $H_\infty$  control is to find a linear feedback control law  $U + B^{-1}\Delta = K\bar{X}$  and a positive definite matrix  $P$  that satisfy the following HJI inequality,

$$P(A+BK) + (A+BK)^T P + \frac{1}{\gamma^2} P D D^T P + C^T C \leq 0$$

Then system (28)-(29) is finite gain  $L_2$ -stable from inputs  $e$  to outputs  $Y$  and the  $L_2$  gain is less than or equal to  $\gamma$ . The preceding inequality is actually equivalent to the following LMIs

$$\begin{bmatrix} AZ + BW + (AZ + BW)^T & D & \frac{1}{\gamma}(CZ)^T \\ D^T & -I & 0 \\ \frac{1}{\gamma}CZ & 0 & -I \end{bmatrix} \leq 0$$

where  $W$  is a unknown matrix and  $Z$  is an unknown positive definite matrix. Once  $W$  and  $Z$  are obtained, the control law can be designed as follows,

$$U = WZ^{-1}\bar{X} - B^{-1}\Delta \quad (30)$$

The control law in Eq. (30) is a feedforward compensation robust controller, which can eliminate the measured disturbances and ensure the system finite gain  $L_2$ -stable from the residual errors  $e$  to the outputs, simultaneously. The controller structure is presented in Fig2.

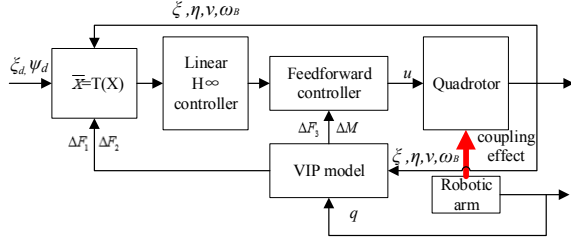


Figure 2. Controller structure of the feedforward compensation linear H $\infty$  controller

#### IV. SIMULATION RESULTS

In order to evaluate the performance of the designed controller, experiments with and without consideration of changes of CoM and inertia tensor are completed and compared through simulations.

##### A. Simulation conditions

The simulations are conducted in Simulink environment of MATLAB R2014a. The solver use ode3 algorithm, and fixed-step size (fundamental sample time) is set to be 0.01s.

The system parameters of the quadrotor used in the simulations are given in Table-I. The inertial parameters and Standard DH parameters of the manipulator are given in Table-II and Table-III, respectively.

In order to simulate the real system, measurement noises, white noise with zero mean and 0.01 standard deviation, are added in all measures. To take influence of system uncertainties and unmodeled into consideration, random signals are also added to translational and rotational dynamics. Specifically, a white noise with zero mean and 1.5 standard deviation for translational dynamic and a white noise with zero mean and 0.5 standard deviation for rotational dynamic.

TABLE I. PARAMETERS OF QUADROTOR

Parameter	Description
$m_Q = 2.43 \text{ kg}$	Mass of quadrotor
$I_{xx} = 0.048 \text{ kg.m}^2$	Inertia on X axis
$I_{yy} = 0.048 \text{ kg.m}^2$	Inertia on Y axis
$I_{zz} = 0.096 \text{ kg.m}^2$	Inertia on Z axis
$C_T = 1.32336 \times 10^{-5}$	Thrust coefficient
$C_Q = 1.069715 \times 10^{-5}$	Torque coefficient
$d = 0.351 \text{ m}$	Distance from rotor to center geometrical.

TABLE II. PARAMETERS OF ROBOTIC ARM

Parameter	Description
$m_1 = 0.08 \text{ kg}$	Mass of link 1
$m_2 = 0.15 \text{ kg}$	Mass of link 2
$m_3 = 0.15 \text{ kg}$	Mass of link 3
$m_4 = 1.4 \text{ kg}$	Mass of link 4
$I_{c1} = 10^{-4} \times \mathbf{1}^* [0.107; 0.107; 0] \text{ kg.m}^2$	Inertia referenced to COM of link 1
$I_{c2} = 10^{-4} \times \mathbf{1}^* [0; 5.25; 5.25] \text{ kg.m}^2$	Inertia referenced to COM of link 2
$I_{c3} = 10^{-3} \times \mathbf{1}^* [0; 5.25; 5.25] \text{ kg.m}^2$	Inertia referenced to COM of link 3
$I_{c4} = 10^{-4} \times \mathbf{1}^* [0; 49; 49] \text{ kg.m}^2$	Inertia referenced to COM of link 4

TABLE III. STANDARD DH PARAMETERS OF ROBOTIC ARM

$I$	$\theta$	$d \text{ (m)}$	$a \text{ (m)}$	$\alpha \text{ (rad)}$	offset (rad)
1	$q_1$	0.1	0	$-\pi/2$	0
2	$q_2$	0	0.205	0	$-\pi/2$
3	$q_3$	0	0.205	0	0
4	$q_4$	0	0.205	0	0

##### B. Simulation results

The parameters of the controller used in simulations are as follows,

$$U = K\bar{X} - B^- \Delta$$

$$K = [K_1, K_2, K_3, K_4]$$

$$K_1 = \begin{bmatrix} 0 & 0 \\ 0 & -70.6412 \\ 70.6412 & 0 \\ 0 & 0 \end{bmatrix} \quad K_2 = I_{4 \times 4} \begin{bmatrix} 161.1103 \\ -95.9374 \\ -95.9374 \\ -3.6300 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 0 & 0 \\ 0 & -37.5922 \\ 37.5922 & 0 \\ 0 & 0 \end{bmatrix} \quad K_4 = I_{4 \times 4} \begin{bmatrix} 14.5643 \\ -7.9043 \\ -7.9043 \\ -0.3160 \end{bmatrix}$$

$$B^- = [O_{4 \times 8} \quad B_{12}^-]$$

$$B_{12}^- = I_{4 \times 4} [4.21, 0.1604, 0.1604, 0.0960]^T$$

During the simulation, the manipulator present no relative motion with the quadrotor. Then, after the quadrotor is hovering steadily, i.e., at the 12s of the simulation, the manipulator moves up with the following joint angle,

$$q_1 = \begin{cases} 0, t < 4\pi \\ \frac{\pi}{3} \sin(2t), t \geq 4\pi \end{cases} \quad q_2, q_3 \text{ or } q_4 = \begin{cases} 0, t < 3\pi \\ \frac{\pi}{3} \sin(t), t \geq 3\pi \end{cases}$$

Let initial states of system be,

$$X_0 = [-0.2, -0.2, -0.4, -0.1, -0.1, -0.1, O_{6 \times 1}]^T$$

and the desired outputs are

$$Y_d = [0, 0, -0.2, 0]^T.$$

The results of simulation are shown as Fig. 2. From Fig. 2, it can be clearly seen that the feedforward enhanced  $H_\infty$  controller (the red line) presents much better performance when relative motion between the manipulator and the quadrotor appears. This is extremely clear for the position response of X and Y.

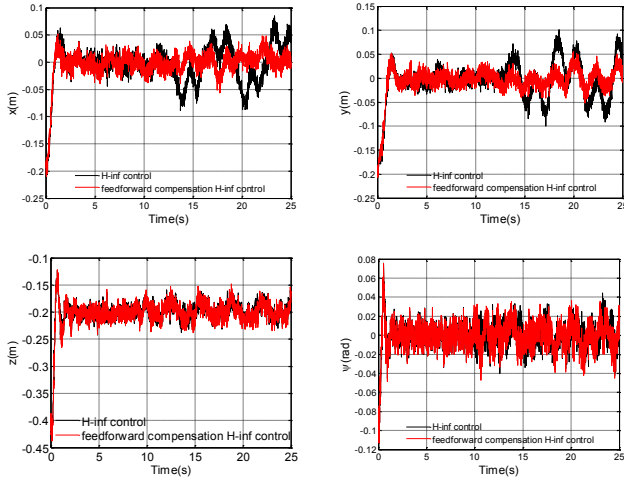


Figure 2. Simulation results (black one is for  $H_\infty$  controller, red one is for feedforward compensation  $H_\infty$  controller).

In order to quantitatively compare the control performance, the following index function is defined,

$$E = \frac{1}{N} \sum_{i=1}^{i=N} \sqrt{[x(i) - x_d(i)]^2}$$

where,  $x(i)$  and  $x_d(i)$  are real and desired value of outputs, respectively. The quantitative results for each controller are given as in Table.

TABLE IV. TRACKING ERROR INDEX E

	$H_\infty$ controller	Feedforward compensation $H_\infty$ controller
x position	0.0308	0.0114
y position	0.0342	0.0160
z position	0.0140	0.0133
yaw angle	0.0129	0.0119

## V. CONCLUSIONS AND FUTURE WORK

In this paper, we present a variable inertia parameters (VIP) model of the aerial manipulator to describe the heavily coupling between the manipulator and the quadrotor due to the relative motion. Based on the VIP model, a feedforward compensation  $H_\infty$  controller is proposed. Simulations are conducted and the results show that the performance of the new proposed controller presents much better performance than a linear  $H_\infty$  controller.

In future's work, our main attention will be in two aspects: firstly, try to find a method that can attenuate the disturbance more effectively; secondly, planning of the manipulator is considered so that the variations of the CoM and the inertia matrix can be limited, i.e., the coupling can be weaken

greatly; finally, experiments in a real system will be conducted to test the real validity and feasibility.

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