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Decentralized Output Feedback Semiglobal Stabilization for a Class of Large-Scale Nonlinear Systems

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***Abstract*— In this paper, the problem of decentralized semiglobal stabilization via output feedback for a class of large-scale nonlinear systems is studied. The systems addressed allow not only nonlinear interconnections but also nonlinear growth with unmeasurable states, which are not able to be stabilized by output feedback compensator in global sense. Based on high-gain observer, decentralized output feedback controllers are presented, which achieves semiglobal asymptotic stabilization of the large-scale nonlinear systems.**

***Index Terms*— Large-scale nonlinear systems, decentralized control, semiglobal stabilization, high-gain observer.**

I. Introduction

How to control a system in the case when only system output is available is an important problem in applications and theories. Not similar to linear systems, this problem is difficult for nonlinear systems for the lack of systematic approach for designing nonlinear observers and the failure of separation principle which may make observers not applicable for output feedback design.

In the last decades, much effort was focused on the problem of output feedback stabilization of nonlinear systems and a lot of significant results have been presented in literatures, for instance, [1], [2], [3], [4], [5], [6]. We can find two schemes in these literatures. First, one imposes some restrictions on nonlinear systems, such as nonlinearities being functions of the measured output or satisfying growth conditions[1], [3], [6], which excludes, of course, a lot of typical nonlinear systems, for example

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= u + x_1^3 + x_2^2 + x_3, \\ y &= x_1. \end{aligned} \tag{1}$$

Second, one allows nonlinear systems to be more general but stabilizes them in a semiglobal sense[2], [4], [5].

A system

$$\begin{aligned} \dot{x} &= f(x,u), \\ y &= g(x), \end{aligned} \tag{2}$$

is said to be semiglobally stabilizable if, for any given compact $K \subset R^n$, there exists a feedback controller such that the closed-loop system is local asymptotically stable at the equilibrium $x = 0$ with K being contained in the domain of attraction. Semiglobal stabilization is peculiar to nonlinear systems, and has received a lot of attention in nonlinear systems researches. [2] studies a general nonlinear separation principle in a semiglobal stabilization sense, in which a important result is presented that, if the equilibrium $x = 0$ of the system (2) is local exponentially and globally stabilizable by a UCO (uniform complete observability) and C^2 state feedback, then it is semiglobally stabilized by a dynamic output feedback. Some separation principles are also given for nonlinear systems with special structures in the semiglobal sense[4], [5].

Large-scale systems can be founded in many practical problems such as power systems, economic systems and traffic networks. Decentralized control is considered as an effective method for large-scale systems for technical and/or economical reasons. There are a lot of systematic results for decentralized stabilization of linear large-scale systems [7], [8], while the progress of decentralized stabilization in large-scale nonlinear systems may seem less significant, especially with decentralized output feedback control. [9], [10], [11], [12] given some results on decentralized state feedback stabilization; [13], [14], [15], [16] addressed decentralized output feedback control problems for large-scale nonlinear systems with special interconnection forms and proposed static output feedback controllers. The decentralized output feedback stabilization based on observer is a challenging problem. [17] studies a class of large-scale nonlinear systems satisfying linear growth condition and gives decentralized observer-based stabilizing controllers by means of singular perturbation method.

In this paper, using semiglobal stabilization notion, we consider the problem of decentralized output feedback stabilization for a class of large-scale nonlinear systems which

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have more general nonlinearity. At the price of sacrificing global stabilization, we present an output feedback compensator achieving semiglobal stabilization of the large-scale nonlinear systems.

II. Problem Statement and Preliminaries

Consider large-scale nonlinear systems of the form

$$\begin{aligned}
\dot{x}_{i,1} &= x_2 + f_1(t, y, x_{i,2}, \cdots, x_{i,n_i}), \\
&\vdots \\
\dot{x}_{i,n_i-1} &= x_{n_i} + f_{n-1}(t, y, x_{i,2}, \cdots, x_{i,n_i}), \\
\dot{x}_{n_i} &= u_i + f_n(t, y, x_{i,2}, \cdots, x_{i,n_i}), \\
y_i &= x_{i,1},\ i = 1, \cdots, N,
\end{aligned} \tag{3}$$

where for each $j = 1, \cdots, n_i, i = 1, \cdots, N$, $x_{i,j} \in R$, $x_i = (x_{i,1}, \cdots, x_{i,n_i})^T \in R^{n_i}$, $y = (y_1, \cdots, y_N)$ are system states and outputs, respectively, $f_{i,j}$ are continuous functions.

We make the following restriction on the systems (3).

Assumption For each $i = 1, \cdots, N$, there exists a continuous nonnegative function $\gamma_i(y, x_{i,2}, \cdots, x_{i,n_i})$ such that, for all $j = 1, \cdots, n_i$, $L \geq 1$, the following inequality holds.

$$\begin{aligned}
&L^{-(j-1)} \left| f_{i,j}(t, y, Lx_{i,2} \cdots, L^{n_i-1} x_{i,n_i}) \right| \\
&\leq \gamma_i(y, x_{i,2}, \cdots, x_{i,n_i})(|x_{i,1}| + |x_{i,2}| + \cdots + |x_{i,n_i}|).
\end{aligned} \tag{4}$$

It is easy to verify that the simple system (1) satisfies this assumption

$$\begin{aligned}
&L^{-2} \left| x_1^3 + (Lx_2)^2 + L^2 x_3 \right| \\
&\qquad \leq (1 + |x_1|^2 + |x_2|)(|x_1| + |x_2| + |x_3|).
\end{aligned} \tag{5}$$

The problem to be addressed in this paper is stated as follows.

Problem of decentralized output feedback semiglobal stabilization: *For every $i = 1, \cdots, N$ and given numbers $K_{i1} > 0$ and $K_{i2} > 0$, find decentralized output feedback control laws*

$$\dot{\bar{x}}_i = \vartheta_i(\bar{x}_i, y_i), u_i = \varpi_i(\bar{x}_i, y_i), \quad i = 1, \cdots, N, \tag{6}$$

$\bar{x}_i \in R^{n_i}$, satisfying $\vartheta_i(0,0) = 0, \varpi_i(0,0) = 0$ such that, the closed-loop system (3)-(6) is asymptotically stable at the origin with domain of attraction containing the compact below

$$\begin{aligned}
&\{x_1 \in R^{n_1} : |x_1| \leq K_{11}\} \times \{\bar{x}_1 \in R^{n_1} : |\bar{x}_1| \leq K_{12}\} \\
&\quad \times \cdots \times \{x_N \in R^{n_N} : |x_N| \leq K_{N1}\} \\
&\qquad \times \{\bar{x}_N \in R^{n_N} : |\bar{x}_N| \leq K_{N2}\}.
\end{aligned}$$

To solve the problem, we introduce two lemmas.

Lemma 1: Consider the following system

$$\dot{z} = F(t, z), \tag{7}$$

where $z \in R^n$, F is continuous with $F(t, 0) = 0$. If there exist a smooth, positive definite, radially unbounded function $V(z)$ and a class-K function α, such that

$$\frac{\partial V}{\partial z} F(t, z) \leq -\alpha(|z|) \tag{8}$$

holds for all z satisfying $V(z) \leq \lambda$, then system (7) is asymptotically stable at the origin with the domain of attraction containing $\{z : V(z) \leq \lambda\}$.

Proof: Please refer to [2] for proof.

Lemma 2: Consider system (7). Assume that there exists a nonsingular linear transformation $\bar{z} = Tz$ changing system (7) into the following form

$$\dot{\bar{z}} = \bar{F}(t, \bar{z}). \tag{9}$$

If there exist a positive number $\bar{\alpha}$ and a smooth, positive definite, radially unbounded function $\bar{V}(\bar{z})$ such that

$$\frac{\partial \bar{V}}{\partial z} \bar{F}(t, \bar{z}) \leq -\bar{\alpha} |\bar{z}|^2 \tag{10}$$

holds for all $\bar{z}$ satisfying $V(\bar{z}) \leq \lambda$, then there exist a positive number α and a smooth, positive definite, radially unbounded function $V(z)$ such that,

$$\frac{\partial V}{\partial z} F(t, z) \leq -\alpha |z|^2 \tag{11}$$

holds for all z satisfying $V(z) \leq \lambda$.

Proof: On applying transformation $\bar{z} = Tz$, we have that

$$\bar{F}(t, \bar{z}) = TF(t, T^{-1}\bar{z}),$$

that is

$$\bar{F}(t, Tz) = TF(t, z).$$

From $\bar{V}(\bar{z})$, we construct

$$V(z) = \bar{V}(Tz).$$

Therefore

$$\begin{aligned}
\frac{\partial V}{\partial z} F(t, z) &= \left. \frac{\partial \bar{V}}{\partial \bar{z}} \right|_{\bar{z}=Tz} TF(t, z) \\
&= \left. \frac{\partial \bar{V}}{\partial \bar{z}} \right|_{\bar{z}=Tz} \bar{F}(t, Tz) = \left. \frac{\partial \bar{V}}{\partial \bar{z}} \bar{F}(t, \bar{z}) \right|_{\bar{z}=Tz}.
\end{aligned}$$

When $V(z) = \bar{V}(Tz) \leq \lambda$, from (10), we get

$$\begin{aligned}
\frac{\partial V}{\partial z} F(t, z) &= \left. \frac{\partial \bar{V}}{\partial \bar{z}} \bar{F}(t, \bar{z}) \right|_{\bar{z}=Tz} \\
&\leq -\bar{\alpha} |Tz|^2 = -\bar{\alpha} z^T T^T Tz.
\end{aligned}$$

Let λ_1 be the minimum eigenvalue of positive definite matrix $T^T T$, it follows that

$$\frac{\partial V}{\partial z} F(z) \leq -\alpha |z|^2$$

with $\alpha = \bar{\alpha} \lambda_1$. This completes the proof. □

III. Main Result

For each $i = 1, \cdots, N$, we introduce the following state estimator for the ith subsystem

$$\begin{aligned}
\dot{\bar{x}}_{i,1} &= \dot{\bar{x}}_{i,2} + L_i a_{i,1}(y_i - \bar{x}_{i,1}), \\
&\vdots \\
\dot{\bar{x}}_{i,n_i-1} &= \dot{\bar{x}}_{i,n_i} + L_i^{n_i-1} a_{i,n_i-1}(y_i - \bar{x}_{i,1}), \\
\dot{\bar{x}}_{i,n_i} &= u_i + L_i^{n_i} a_{i,n_i}(y_i - \bar{x}_{i,1}),
\end{aligned} \tag{12}$$

where $L_i \geq 1$ determined later, $a_{i,1}, \cdots, a_{i,n_i}$ are chosen such that the following polynomial

$$p_i(\lambda) = \lambda^{n_i} + a_{i,1}\lambda^{n_i-1} + \cdots + a_{i,n_i-1}\lambda + a_{i,n_i}$$

has all the eigenvalues with negative real part,

Theorem 3: Consider system (3). Under the assumption , the problem of decentralized output feedback semiglobal stabilization is solvable.

Proof: Let us introduce the new variables

$$e_{i,j} = \frac{x_{i,j} - \bar{x}_{i,j}}{L_i^{j-1}}, \quad \tilde{x}_{i,j} = \frac{\bar{x}_{i,j}}{L_i^{j-1}}, \quad i = 1, \cdots, N, \; j = 1, \cdots, n_i, \tag{13}$$

where L_i is the same as (12). In new coordinates, composite system (3)-(12) is of the form

$$\begin{aligned}
\dot{e}_{i,1} &= L_i e_{i,2} - L_i a_{i,1} e_{i,1} + f_{i,1}\big(t, y, L_i(e_{i,2} + \tilde{x}_{i,2}), \cdots, L_i^{n_i-1}(e_{i,n} + \tilde{x}_{i,n})\big), \\
\dot{e}_{i,2} &= L_i e_{i,3} - L_i a_{i,2} e_{i,1} + L_i^{-1} f_{i,2}\big(t, y, L_i(e_{i,2} + \tilde{x}_{i,2}), \cdots, L_i^{n_i-1}(e_{i,n} + \tilde{x}_{i,n})\big), \\
&\vdots \\
\dot{e}_{i,n_i} &= -L_i a_{i,n_i} e_{i,1} + L_i^{-(n_i-1)} f_{i,n_i}\big(t, y, L_i(e_{i,2} + \tilde{x}_{i,2}), \cdots, L_i^{n_i-1}(e_{i,n} + \tilde{x}_{i,n})\big), \\
\dot{\tilde{x}}_{i,1} &= L_i \tilde{x}_{i,2} + L_i a_{i,1} e_{i,1}, \\
\dot{\tilde{x}}_{i,2} &= L_i \tilde{x}_{i,3} + L_i a_{i,2} e_{i,1}, \\
&\vdots \\
\dot{\tilde{x}}_{i,n_i} &= \frac{1}{L^{n_i-1}} u_i + L_i a_{i,n} e_{i,1}, \\
y_i &= e_{i,1} + \tilde{x}_{i,1}, \; i = 1, \cdots, N.
\end{aligned} \tag{14}$$

Choose $b_{i,n_i}, b_{i,n_i-1}, \cdots, b_{i,1}$ such that they are the coefficients of a Hurwitz polynomial

$$q_i(\lambda) = \lambda^{n_i} + b_{i,n_i}\lambda^{n_i-1} + \cdots + b_{i,2}\lambda + b_{i,1}$$

and set

$$u_i = -L_i^{n_i}(b_{i,1}\tilde{x}_{i,1} + b_{i,2}\tilde{x}_{i,2} + \cdots + b_{i,n_i}\tilde{x}_{i,n_i}). \tag{15}$$

By introducing new variables

$$\zeta_i = (e_{i,1}, \cdots, e_{i,n_i}, \tilde{x}_{i,1}, \cdots, \tilde{x}_{i,n_i})^T, \tag{16}$$

$i = 1, \cdots, N$, and letting

$$\zeta = (\zeta_1^T, \cdots, \zeta_N^T)^T$$

the closed-loop system (14)-(15) can be rewritten as

$$\dot{\zeta}_i = L_i D_i \zeta_i + F_i(t, L_i, \zeta), i = 1, \cdots, N, \tag{17}$$

where

$$\begin{aligned}
\tilde{f}_{i,j} &= L_i^{-(j-1)} f_{i,j}\big(t, e_{i,1} + \tilde{x}_{i,1}, \cdots, e_{N,1} + \tilde{x}_{N,1}, L_i(e_{i,2} + \tilde{x}_{i,2}), \cdots, L_i^{n_i-1}(e_{i,n} + \tilde{x}_{i,n})\big) \\
F_i(t, L_i, \zeta) &= (\tilde{f}_{i,1}, \tilde{f}_{i,2}, \cdots, \tilde{f}_{i,n_i} 0, \cdots, 0)^T
\end{aligned}$$

$$A_i = \begin{pmatrix} -a_{i,1} & & & \\ \vdots & & I & \\ -a_{i,n_i-1} & & & \\ -a_{i,n_i} & 0 & \cdots & 0 \end{pmatrix},$$

$$B_i = \begin{pmatrix} 0 & & & \\ \vdots & & I & \\ 0 & & & \\ -b_{i,1} & -b_{i,2} & \cdots & -b_{i,n_i} \end{pmatrix},$$

$$C_i = \begin{pmatrix} a_{i,1} & 0 & 0 & ... & 0 \\ a_{i,2} & 0 & 0 & ... & 0 \\ ... & ... & ... & ... & 0 \\ a_{i,n_i} & 0 & 0 & ... & 0 \end{pmatrix}, \; D_i = \begin{pmatrix} A_i & 0 \\ C_i & B_i \end{pmatrix}.$$

Thanks to the selection of $a_{i,1}, \cdots, a_{i,n_i-1}, a_{i,n_i}$and $b_{i,1}, \cdots, b_{i,n_i-1}, b_{i,n_i}$, A_i, B_i and therefore D_i are Hurwitz matrix.

From the assumption, we have

$$\begin{aligned}
&\big|L_i^{-(j-1)} f_{i,j}\big(t, e_{i,1} + \tilde{x}_{i,1}, \cdots, e_{N,1} + \tilde{x}_{N,1}, L_i(e_{i,2} + \tilde{x}_{i,2}), \cdots, L_i^{n_i-1}(e_{i,n_i} + \tilde{x}_{i,n_i})\big)\big| \\
&\leq (|e_{i,1} + \tilde{x}_{i,1}| + \cdots + |e_{i,n_i} + \tilde{x}_{i,n_i}|) \cdot \gamma_i(e_{1,1} + \tilde{x}_{1,1}, \cdots, e_{N,1} + \tilde{x}_{N,1}, e_{i,2} + \tilde{x}_{i,2}, \cdots, e_{i,n_i} + \tilde{x}_{i,n_i}) \\
&\leq (|e_1| + \cdots + |e_n| + |\tilde{x}_1| + \cdots + |\tilde{x}_n|) q_i(\zeta),
\end{aligned}$$

where $q_i(\zeta)$ is a suitable continuous nonnegative function. So that, a continuous nonnegative function $R_i(\zeta)$ exists such that

$$|F_i(t, L_i, \zeta)| \leq |\zeta| \, R_i(\zeta).$$

Define a compact subset of $R^{n_i+n_i}$ as follows

$$\Lambda_i = \big\{(x_i^T, \bar{x}_i^T)^T : x_i \in R^{n_i}, \bar{x}_i \in R^{n_i}, |x_i| \leq K_{i1} + K_{i2}, |\bar{x}_i| \leq K_{i2}\big\},$$

Owing to D_i being a stable matrix, a positive definite matrix P_i exists such that

$$D_i^T P_i + P_i D_i = -I.$$

Let λ_i be the maximum eigenvalue of P_i and let

$$\mu_i = \max\left\{\lambda_i \zeta_i^T \zeta_i : \zeta_i \in \Lambda_i\right\}.$$

Consider the Lyapunov function

$$\bar{W}_i(\zeta_i) = \zeta_i^T P_i \zeta_i.$$

Due to the definition of μ_i, we have that

$$\bar{W}_i(\zeta_i) \leq \zeta_i^T P_i \zeta_i \leq \mu_i, \text{ for all } \zeta_i \in \Lambda_i. \tag{18}$$

That is

$$\{\zeta_i : \bar{W}_i(\zeta_i) \leq \mu_i\} \supset \Lambda_i$$

Define

$$\bar{W}(\zeta) = \bar{W}_1(\zeta_1) + \cdots + \bar{W}_N(\zeta_N), \tag{19}$$

and let

$$\mu = \mu_1 + \cdots + \mu_N.$$

We get

$$\{\zeta : \bar{W}(\zeta) \le \mu\} \supset \Lambda_i \times \cdots \times \Lambda_N.$$

Thanks to the continuity of $R_i(\zeta)$, there exists a real number $C_i > 0$ such that

$$R_i(\zeta) \le C_i, \tag{20}$$

for all ζ satisfying $\bar{W}(\zeta) \le \mu$, $i = 1, \cdots, N$.

Computing the time derivative of $\bar{W}(\zeta)$ along the solutions of the system (17), we get that

$$\begin{aligned}\dot{\bar{W}} &= \frac{\partial \bar{W}_1}{\partial \zeta_1}(L_1 D_1 \zeta_1 + F_1(t, L_1, \zeta)) \\ &\quad + \cdots + \\ &\quad \frac{\partial \bar{W}_N}{\partial \zeta_N}(L_N D_N \zeta_N + F_N(t, L_N, \zeta)) \\ &\le -L_1 |\zeta_1|^2 + 2\|P_1\| |\zeta_1|^2 R_1(\zeta) \\ &\quad + \cdots + \\ &\quad -L_N |\zeta_N|^2 + 2\|P_N\| |\zeta_N|^2 R_N(\zeta).\end{aligned}$$

Using (20) yields

$$\begin{aligned}\dot{\bar{W}} &\le (-L_1 + 2\|P_1\| C_1) |\zeta_1|^2 \\ &\quad + \cdots + \\ &\quad (-L_N + 2\|P_N\| C_N) |\zeta_N|^2.\end{aligned}$$

Choosing

$$L_i^* = 2C_i \|P_i\| + 1, \tag{21}$$

yields that, for all ζ satisfying $\bar{W}(\zeta) \le \mu$, it follows that

$$\dot{\bar{W}} \le -|\zeta|^2.$$

Now return to the original coordinates and consider the composite system (3)-(12) and set

$$\begin{aligned}u_i = -L_i^{*n_i}\big(&b_{i,1}\bar{x}_{i,1} + b_{i,2}L_i^{*-1}\bar{x}_{i,2} \\ &+ \cdots + b_{i,n_i}L_i^{*-(n_i-1)}\bar{x}_{i,n_i}\big).\end{aligned} \tag{22}$$

The closed-loop system made of (3), (12) and (22) can be changed into system (17) with nonsingular transformation (13). Let

$$\begin{aligned}&W_i(x_i, \bar{x}_i) \\ &= W_i(x_{i,1}, \cdots, x_{i,n_i}, \bar{x}_{i,1}, \cdots, \bar{x}_{i,n_1}) \\ &= \bar{W}_i\left(x_{i,1} - \bar{x}_{i,1}, \cdots, \frac{x_{i,n_i} - \bar{x}_{i,n_i}}{L_i^{*n_i-1}}, \bar{x}_{i,1}, \cdots, \frac{\bar{x}_{i,n_i}}{L_i^{*n_i-1}}\right),\end{aligned}$$

and let

$$W(x_1, \bar{x}_1, \cdots, x_N, \bar{x}_N) = W_1(x_1, \bar{x}_1) + \cdots + W_N(x_N, \bar{x}_N).$$

From the conclusion and proof of Lemma 2, the time derivative of W along the closed-loop system made of (3), (12) and (22) satisfies

$$\dot{W} \le -\alpha \left|(x_1^T, \bar{x}_1^T, \cdots, x_N^T, \bar{x}_N^T)^T\right|^2, \tag{23}$$

for all $x_1, \bar{x}_1, \cdots, x_N, \bar{x}_N$ satisfying

$$W(x_1, \bar{x}_1, \cdots, x_N, \bar{x}_N) \le \mu,$$

where α is a positive number.

In what follows, we will prove that all points in the compact set

$$\begin{aligned}&\{x_1 \in R^{n_1} : |x_1| \le K_{11}\} \times \{\bar{x}_1 \in R^{n_1} : |\bar{x}_1| \le K_{12}\} \\ &\quad \times \cdots \times \{x_N \in R^{n_N} : |x_N| \le K_{N1}\} \\ &\quad\quad \times \{\bar{x}_N \in R^{n_N} : |\bar{x}_N| \le K_{N2}\}.\end{aligned}$$

satisfy

$$W(x_1, \bar{x}_1, \cdots, x_N, \bar{x}_N) \le \mu.$$

It is sufficient to show, for all $i = 1, \cdots, N$,

$$\begin{aligned}&\{(x_i^T, \bar{x}_i^T)^T : W_i(x_i, \bar{x}_i) \le \mu_i\} \\ &\supseteq \{x_i \in R^{n_i} : |x_i| \le K_{i1}\} \times \{\bar{x}_i \in R^{n_i} : |\bar{x}_i| \le K_{i2}\}.\end{aligned} \tag{24}$$

From the definition of $\bar{W}$ and λ_i, we have

$$\begin{aligned}&W_i(x_{i,1}, \cdots, x_{i,n_i}, \bar{x}_{i,1}, \cdots, \bar{x}_{i,n_i}) \\ &= \bar{W}_i\left(x_{i,1} - \bar{x}_{i,1}, \cdots, \frac{x_{i,n_i} - \bar{x}_{i,n_i}}{L_i^{*n_i-1}}, \bar{x}_{i,1}, \cdots, \frac{\bar{x}_{i,n_i}}{L_i^{*n_i-1}}\right) \\ &\le \lambda_i\Big[(x_{i,1} - \bar{x}_{i,1})^2 + \cdots + \big(L_i^{*-(n_i-1)}(x_{i,n_i} - \bar{x}_{i,n_i})\big)^2 \\ &\quad + \bar{x}_{i,1}^2 + \cdots + (L_i^{*-(n_i-1)}\bar{x}_{i,n_i})^2\Big] \\ &\le \lambda_i\big[(x_{i,1} - \bar{x}_{i,1})^2 + \cdots + (x_{i,n_i} - \bar{x}_{i,n_i})^2 \\ &\quad + \bar{x}_{i,1}^2 + \cdots + \bar{x}_{i,n_i}^2\big].\end{aligned}$$

So, for all $(x_{i,1}, \cdots, x_{i,n_i})$ and $(\bar{x}_{i,1}, \cdots, \bar{x}_{i,n_i})$ satisfying $|(x_{i,1}, \cdots, x_{i,n_i})^T| \le K_{i1}$ and $|(\bar{x}_{i,1}, \cdots, \bar{x}_{i,n_i})^T| \le K_{i2}$ respectively, it is straightforward to show that

$$|(x_{i,1} - \bar{x}_{i,1}, \cdots, x_{i,n_i} - \bar{x}_{i,n_i})^T| \le K_{i1} + K_{i2},$$

and then

$$(x_{i,1} - \bar{x}_{i,1}, \cdots, x_{i,n_i} - \bar{x}_{i,n_i}, \bar{x}_{i,1}, \cdots, \bar{x}_{i,n_i})^T \in \Lambda_i.$$

Using the definition of μ_i, we obtain that

$$\begin{aligned}&W_i(x_{i,1}, \cdots, x_{i,n_i}, \bar{x}_{i,1}, \cdots, \bar{x}_{i,n_i}) \\ &\quad \le \lambda_i\big[(x_{i,1} - \bar{x}_{i,1})^2 + \cdots + (x_{i,n_i} - \bar{x}_{i,n_i})^2 \\ &\quad\quad + \bar{x}_{i,1}^2 + \cdots + \bar{x}_{i,n_i}^2\big] \\ &\quad \le \mu_i,\end{aligned}$$

that is, (24) holds.

To sum up, we have that

$$\dot{W} \le -\alpha \left|(x_1^T, \bar{x}_1^T, \cdots, x_N^T, \bar{x}_N^T)^T\right|^2,$$

for all $x_1, \bar{x}_1, \cdots, x_N, \bar{x}_N$ satisfying

$$W(x_1, \bar{x}_1, \cdots, x_N, \bar{x}_N) \le \mu,$$

and set

$$\{(x_1^T, \bar{x}_1^T, \cdots, x_N^T, \bar{x}_N^T)^T : W(x_1, \bar{x}_1, \cdots, x_N, \bar{x}_N) \le \mu\}$$

contains

$$\begin{aligned}&\{x_1 \in R^{n_1} : |x_1| \le K_{11}\} \times \{\bar{x}_1 \in R^{n_1} : |\bar{x}_1| \le K_{12}\} \\ &\quad \times \cdots \times \{x_N \in R^{n_N} : |x_N| \le K_{N1}\} \\ &\quad\quad \times \{\bar{x}_N \in R^{n_N} : |\bar{x}_N| \le K_{N2}\}.\end{aligned}$$

By Lemma 1, the proof is completed. □

Remark 1: We see, from the design procedure, that the gain L_i, $i = 1, \cdots, N$, depends not only on the given compact subsets but also on the nonlinearities of systems. Rapid nonlinear growth may make these gains too large to be used in applications.

IV. Conclusion

In semiglobal stabilization sense, we consider the problem of decentralized output feedback stabilization of large-scale nonlinear systems under a moderate assumption. The design approach here allows nonlinear systems to possess nonlinear growth with unmeasurable states, therefore, is may be applicable to more general large-scale nonlinear systems.

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