An LMI Approach to Stability of Systems With Severe Time-Delay

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***Abstract*—This note describes the stability problems of uncertain systems with arbitrarily time-varying and severe time-delay. Using new Lyapunov–Krasovskii functionals, less conservative stability conditions are obtained for such systems. The results are illustrated using the numerical examples based on simple linear matrix inequalities.**

***Index Terms*—Linear matrix inequality (LMI), stability, time-delay systems, time-varying delay.**

I. Introduction

In many cases, time delay is a source of instability [1]. Therefore, severely time-delayed systems may face great challenges in achieving desired stability. Recently, improved performances [2], [5]–[10] have been reported by using Lyapunov–Krasovskii theorems and linear matrix ineqaulity (LMI) techniques [1], [4]. However, proper selection of Lyapunov–Krasovskii functional is crucial for deriving stability conditions [11]. Different Lyapunov–Krasovskii functionals may result in different stability conditions with different conservatism and advantage. Many methods have been proposed to reduce the conservatism for this kind of time-domain approaches.

For instances, Park proposed a new upper bound for the inner product of two vectors [12], and less conservative results have been reported when applying this new bound [3], [13]. Other authors proposed an equivalent transformation for the time-delay systems (often referred to as a descriptor system) [3], and enhanced system performances have been demonstrated by applying this transformation. Comparing different model transformations were also performed to show the advantages of the descriptor system model [13]. One of the difficulties in applying Lyapunov–Krasovskii methods is, however, the lack of efficient algorithms for constructing the Lyapunov–Krasovskii functionals. In general, the use of reduced functionals may result in conservatism. To solve this problem, a procedure for the construction of full-size quadratic functionals for the linear time-delay systems is developed [14]. To improve the results, some authors adopted an approach to discretize Lyapunov–Krasovskii functionals [15]. To summarize, the conservatism can be reduced by: 1) the development of new bounding techniques for the inner product of involved cross-terms, 2) the transformation of the original system to the one with distributed time-delay, and 3) the construction of new Lyapunov–Krasovskii functionals with a proper distribution of the time delays. The first two methods have been extensively discussed in the publications, and the third one is elaborated in this note. The authors have also noted that most of the existing results obtained using Laypunov–Krasovskii stability theorems for the systems with severely time-varying delays require constraints on the time derivative of the delays.

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In this note, new Lyapunov–Krasovskii functionals are proposed to obtain less conservative stability conditions for a class of uncertain systems with arbitrarily time-varying delay. The results are less conservative and more generic than the existing ones.

II. Problem Statements

Consider the following uncertain system with a time-varying delay:

$$\dot{x}(t) = (A + \Delta A(t))\, x(t) + (B + \Delta B(t))\, x(t - h(t)) \quad (t \geq 0) \tag{1a}$$

$$x(t) = \phi(t), \qquad t \in [-h, 0] \tag{1b}$$

where $x(t) \in \Re^n$; $\phi(t)$ is a smooth vector-valued initial function defined in the Banach space $C[-h,0]$ of smooth functions $\psi : [-h,0] \mapsto \Re^n$ with $\|\psi\|_\infty := \sup_{-h \leq \eta \leq 0} \psi(\eta)$; A and B are known real constant matrices with appropriate dimensions; $\Delta A(t)$ and $\Delta B(t)$ are time-varying uncertainties, and are assumed to have the forms of

$$\Delta A(t) = DF(t)E \quad \Delta B(t) = GF_1(t)H \tag{2a}$$

where D, E, G, H are constant matrices of appropriate dimensions and

$$F(t)F^T(t) \leq I \quad F_1(t)F_1^T(t) \leq I \quad \forall t \tag{2b}$$

$h(t)$ denotes the time-varying delay, and is assumed to satisfy either A1) or A2) as

$$\text{A1)}\ 0 \leq h(t) \leq h, \qquad \dot{h}(t) \leq d \tag{3a}$$

$$\text{A2)}\ 0 \leq h(t) \leq h. \tag{3b}$$

The following lemma is used [17].

Lemma 1: Let U, V, and F be real matrices of appropriate dimensions with $FF^T \leq I$, then for any scalar $\varepsilon > 0$, we have $UFV + V^T F^T U^T \leq \varepsilon^{-1} UU^T + \varepsilon V^T V$.

III. Main Results

First, (1a) can be rewritten as

$$\begin{aligned} \dot{x}(t) &= y(t) \\ y(t) &= (A + \Delta A)x(t) + (B + \Delta B)x(t - h(t)) \end{aligned} \tag{4}$$

with the identical initial conditions as expressed in (1b). It is noted that (4) is completely equivalent to (1a) [13].

Theorem 1: Assume the time delay satisfies A1). If there exist $P > 0$, $Q > 0$, P_1, P_2, $X_{\rm ij}$ (i,j = 1,2,3), and $\varepsilon_i > 0$ $(i = 1,2,3,4)$, such that the LMI, shown in (5) at the bottom of the next page, holds, and

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix} > 0 \tag{6}$$

where

$$\begin{aligned} W_1 &= A^T P_1 + P_1^T A + hX_{11} + X_{13} + X_{13}^T \\ &\quad + Q + (\varepsilon_1 + \varepsilon_3)E^T E \\ W_2 &= P_1^T B + hX_{12} - X_{13} + X_{23}^T \\ W_3 &= hX_{22} - X_{23} - X_{23}^T - (1-d)Q + (\varepsilon_2 + \varepsilon_4)H^T H. \end{aligned}$$

Then system (1a) is asymptotically stable, dependent on h and d.

Proof: The Lyapunov–Krasovskii functionals are constructed as follows:

$$V = V_1 + V_2 + V_3 + V_4 \tag{7}$$

where

$$V_1 = x^T(t)Px(t) \tag{8}$$

$$V_2 = \int_0^h (h-\sigma)\dot{x}^T(t-\sigma)X_{33}\dot{x}(t-\sigma)d\sigma \tag{9}$$

$$V_3 = \int_0^t \int_{\sigma-h(\sigma)}^{\sigma} e^T X e ds d\sigma \tag{10}$$

with $e = \begin{bmatrix} x(\sigma) \\ x(\sigma-h(\sigma)) \\ \dot{x}(s) \end{bmatrix}, X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix} = X^T > 0$

$$V_4 = \int_{t-h(t)}^{t} x^T(s)Qx(s)ds. \tag{11}$$

Now, we consider the derivative of V along the trajectories of (1a). For the first term V_1, we have $\forall P_1, P_2$

$$\begin{aligned}\dot{V}_1 &= 2x^T(t)P\dot{x}(t) = 2x^T(t)Py(t)\\
&= 2x^T(t)Py(t) + 2\left[x^T(t)P_1^T + y^T(t)P_2^T\right]\\
&\quad \cdot[-y(t) + (A+\Delta A)x(t) + (B+\Delta B)x(t-h(t))]\\
&= 2x^T(t)Py(t) + 2x^T(t)P_1^T Ax(t)\\
&\quad + 2x^T(t)P_1^T Bx(t-h(t)) + 2x^T(t)P_1^T \Delta Ax(t)\\
&\quad + 2x^T(t)P_1^T \Delta Bx(t-h(t)) - 2x^T(t)P_1^T y(t)\\
&\quad - 2y^T(t)P_2^T y(t) + 2y^T(t)P_2^T Ax(t)\\
&\quad + 2y^T(t)P_2^T Bx(t-h(t)) + 2y^T(t)P_2^T \Delta Ax(t)\\
&\quad + 2y^T(t)P_2^T \Delta Bx(t-h(t))\\
&\le 2x^T(t)Py(t) + 2x^T(t)P_1^T Ax(t)\\
&\quad + 2x^T(t)P_1^T Bx(t-h(t)) + \varepsilon_1^{-1}x^T(t)P_1^T DD^T P_1 x(t)\\
&\quad + \varepsilon_1 x^T(t)E^T Ex(t) + \varepsilon_2^{-1}x^T(t)P_1^T GG^T P_1 x(t)\\
&\quad + \varepsilon_2 x^T(t-h(t))H^T Hx(t-h(t)) - 2x^T(t)P_1^T y(t)\\
&\quad - 2y^T(t)P_2^T y(t) + 2y^T(t)P_2^T Ax(t)\\
&\quad + 2y^T(t)P_2^T Bx(t-h(t)) + \varepsilon_3^{-1}y^T(t)P_2^T DD^T P_2 y(t)\\
&\quad + \varepsilon_3 x^T(t)E^T Ex(t) + \varepsilon_4^{-1}y^T(t)P_2^T GG^T P_2 y(t)\\
&\quad + \varepsilon_4 x^T(t-h(t))H^T Hx(t-h(t))\end{aligned}$$

In the second and third equality, the representation (4) is used, and in the first inequality, Lemma 1 is used. Equation (9) can be described as:

$$\begin{aligned}V_2 &= \int_{t-h}^{t} (h+s-t)\dot{x}^T(s)X_{33}\dot{x}(s)ds\\
&= \int_{t-h}^{t} (h+s)\dot{x}^T(s)X_{33}\dot{x}(s)ds - t\\
&\quad \times \int_{t-h}^{t} \dot{x}^T(s)X_{33}\dot{x}(s)ds.\end{aligned} \tag{12}$$

Then, from (12), we have

$$\begin{aligned}\dot{V}_2 &= (h+t)\dot{x}^T(t)X_{33}\dot{x}(t) - t\dot{x}^T(t-h)X_{33}\dot{x}(t-h)\\
&\quad - t\left[\dot{x}^T(t)X_{33}\dot{x}(t) - \dot{x}^T(t-h)X_{33}\dot{x}(t-h)\right]\\
&\quad - \int_{t-h}^{t} \dot{x}^T(s)X_{33}\dot{x}(s)ds\\
&= h\dot{x}^T(t)X_{33}\dot{x}(t) - \int_{t-h}^{t} \dot{x}^T(s)X_{33}\dot{x}(s)ds\\
&= hy^T(t)X_{33}y(t) - \int_{t-h}^{t} y^T(s)X_{33}y(s)ds\end{aligned}$$

The derivative for (10) is

$$\begin{aligned}\dot{V}_3 &= h(t)\begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}\\
&\quad + 2x^T(t)X_{13}x(t) - 2x^T(t)X_{13}x(t-h(t))\\
&\quad + 2x^T(t-h(t))X_{23}x(t) - 2x^T(t-h(t))X_{23}x(t-h(t))\\
&\quad + \int_{t-h(t)}^{t} \dot{x}^T(s)X_{33}\dot{x}(s)ds\\
&\le hx^T(t)X_{11}x(t) + 2hx^T(t)X_{11}x(t-h(t))\\
&\quad + hx^T(t-h(t))X_{11}x(t-h(t)) + 2x^T(t)X_{13}x(t)\\
&\quad - 2x^T(t)X_{13}x(t-h(t)) + 2x^T(t-h(t))X_{23}x(t)\\
&\quad - 2x^T(t-h(t))X_{23}x(t-h(t)) + \int_{t-h}^{t} y^T(s)X_{33}y(s)ds\end{aligned}$$

In the aforementioned inequality, Assumption A1 is used. The derivative of V_4 (11) satisfies

$$\dot{V}_4 \le x^T(t)Qx(t) - (1-d)x^T(t-h(t))Qx(t-h(t)).$$

$$\begin{bmatrix} W_1 & P - P_1^T + A^T P_2 & W_2 & P_1^T D & P_1^T G & 0 & 0 \\ * & -P_2 - P_2^T + hX_{33} & P_2^T B & 0 & 0 & P_2^T D & P_2^T G \\ * & * & W_3 & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_3 I & 0 \\ * & * & * & * & * & * & -\varepsilon_4 I \end{bmatrix} < 0 \tag{5}$$

Finally, we have

$$
\begin{aligned}
\dot{V} \leq\, & 2x^T(t)Py(t)+2x^T(t)P_1^TAx(t)\\
&+2x^T(t)P_1^TBx(t-h(t))+\varepsilon_1^{-1}x^T(t)P_1^TDD^TP_1x(t)\\
&+\varepsilon_1x^T(t)E^TEx(t)+\varepsilon_2^{-1}x^T(t)P_1^TGG^TP_1x(t)\\
&+\varepsilon_2x^T(t-h(t))H^THx(t-h(t))-2x^T(t)P_1^Ty(t)\\
&-2y^T(t)P_2^Ty(t)+2y^T(t)P_2^TAx(t)\\
&+2y^T(t)P_2^TBx(t-h(t))+\varepsilon_3^{-1}y^T(t)P_2^TDD^TP_2y(t)\\
&+\varepsilon_3x^T(t)E^TEx(t)+\varepsilon_4^{-1}y^T(t)P_2^TGG^TP_2y(t)\\
&+\varepsilon_4x^T(t-h(t))H^THx(t-h(t))\\
&+hy^T(t)X_{33}y(t)-\int_{t-h}^{t}y^T(s)X_{33}y(s)d\\
&+hx^T(t)X_{11}x(t)+2hx^T(t)X_{12}x(t-h(t))\\
&+hx^T(t-h(t))X_{22}x(t-h(t))+2x^T(t)X_{13}x(t)\\
&-2x^T(t)X_{13}x(t-h(t))+2x^T(t-h(t))X_{23}x(t)\\
&-2x^T(t-h(t))X_{23}x(t-h(t))+\int_{t-h}^{t}y^T(s)x_{33}y(s)ds\\
&+x^T(t)Qx(t)-(1-d)x^T(t-h(t))Qx(t-h(t))\\
=\,&\begin{bmatrix}x(t)\\ y(t)\\ x(t-h(t))\end{bmatrix}^T\Omega\begin{bmatrix}x(t)\\ y(t)\\ x(t-h(t))\end{bmatrix}
\end{aligned}
\tag{13}
$$

where

$$
\Omega=\begin{bmatrix}
\Xi & P-P_1^T+A^TP_2 & \begin{matrix}P_1^TB+hX_{12}\\ -X_{13}+X_{23}^T\end{matrix}\\
* & \begin{matrix}-P_2-P_2^T+hX_{33}\\ +\varepsilon_3^{-1}P_2^TDD^TP_2\\ +\varepsilon_4^{-1}P_2^TGG^TP_2\end{matrix} & P_2^TB\\
* & * & \begin{matrix}hX_{22}-X_{23}-X_{23}^T\\ -(1-d)Q\\ +(\varepsilon_2+\varepsilon_4)H^TH\end{matrix}
\end{bmatrix}.
$$

$$
\begin{aligned}
\Xi =\, & A^TP_1+P_1^TA+hX_{11}+X_{13}+X_{13}^T+Q\\
&+\varepsilon_1^{-1}P_1^TDD^TP_1+\varepsilon_2^{-1}P_1^TGG^TP_1+(\varepsilon_1+\varepsilon_3)E^TE
\end{aligned}
$$

Using the Schur complement, we find that $\dot{V}$ is negative as long as the inequalities (5) and (6) hold, which implies that system (1a) is asymptotically stable. This completes the proof.

Remark 1: For the functionals adopted in Theorem 1, V_1 and V_4 have been used in many publications [2], V_2 is inspired by [14] and [15] where delay-dependent quadratic functions are constructed, and V_3 is partly inspired by the work in [16] where a receding horizon control problem is considered. After the submission of this note, the authors learned of Lee's publication [20] in which a similar functional to V_3 was used in his work to deal with a constant time-delay case.

TABLE I
FOR EXAMPLE 1

Methods	h (d is unknown)	h (d=0.1)	h (d=2)	h ($\dot{h}(t)$ =0)
Niculescu *et.al.*[5]	0.3440			
Su [6]	0.4045			
Li and de Souza [7]	0.7218			
Kim [8]		0.9447	Not defined	1
Yue and Won [9]		0.9723	Not defined	1
Park [12]				4.3588
Fridman and Shaked [3]				4.47
Our result	0.9999	3.6040	0.9999	4.4721

Remark 2: Most of the existing delay-derivative-dependent conditions [8], [9], [13] for the stability of systems with severely time-varying delay generally require a constraint of $d < 1$. Instead, the conditions provided in Theorem 1 hold for all $d \in \Re$. Furthermore, instead of adopting Park's inequality in the derivation of Theorem 1, as was used in published results [3], [12], and [13], the authors used only the Lyapunov–Krasovskii functionals. It shall be noted that by properly distributing the time delay in the Lyapunov–Krasovskii functionals, less conservative stability conditions can be obtained. In addition, if d is set to be zero in (5), the conditions in Theorem 1 correspond to the constant delay case.

If the Lyapunov–Krasovskii functionals are chosen to be $V = V_1 + V_2 + V_3$, as described in the proof of Theorem 1, then delay-derivative-free stability conditions for system (1a) would follow.

Corollary 1: Assume the time delay satisfies A2. If there exist $P > 0$, P_1, P_2, X_{ij} $(i, \mathrm{j} = 1, 2, 3)$, and $\varepsilon_i > 0$ $(\mathrm{i} = 1, 2, 3, 4)$, such that the LMI shown in (14) at the bottom of the page, holds, and

$$
\begin{bmatrix} X_{11} & X_{12} & X_{13}\\ * & X_{22} & X_{23}\\ * & * & X_{33}\end{bmatrix} > 0 \tag{15}
$$

where

$$
\begin{aligned}
W_4 &= A^TP_1+P_1^TA+hX_{11}+X_{13}+X_{13}^T+(\varepsilon_1+\varepsilon_3)E^TE\\
W_2 &= P_1^TB+hX_{12}-X_{13}+X_{23}^T\\
W_5 &= hX_{22}-X_{23}-X_{23}^T+(\varepsilon_2+\varepsilon_4)H^TH.
\end{aligned}
$$

Then, (1a) is delay-dependently asymptotically stable, and its stability is independent of d.

Proof: From the proof of Theorem 1, this corollary follows immediately. This completes the proof.

Remark 3: To the best of our knowledge, the Razumikhin-type theorem-based method has been the only one capable of coping with case A2), while only a few results for case A2) have been obtained using

$$
\begin{bmatrix}
W_4 & P-P_1^T+A^TP_2 & W_2 & P_1^TD & P_1^TG & 0 & 0\\
* & -P_2-P_2^T+hX_{33} & P_2^TB & 0 & 0 & P_2^TD & P_2^TG\\
* & * & W_5 & 0 & 0 & 0 & 0\\
* & * & * & -\varepsilon_1I & 0 & 0 & 0\\
* & * & * & * & -\varepsilon_2I & 0 & 0\\
* & * & * & * & * & -\varepsilon_3I & 0\\
* & * & * & * & * & * & -\varepsilon_4I
\end{bmatrix} < 0. \tag{14}
$$

TABLE II
FOR EXAMPLE 2

Method	Su[6]	Chen [19]	Gu [18]	Li [7]	Kim [8]	Our result
h	0.3188	0.3570	0.3809	0.4428	0.5351	0.8522

Lyapunov–Krasovskii functionals [13]. Instead, the results for case A2) are obtained by using the new Lyapunov–Krasovskii functionals in Corollary 1.

IV. NUMERICAL EXAMPLES

Consider the nominal system for (1) with

$$A = \begin{bmatrix} -2.0 & 0 \\ 0 & -0.9 \end{bmatrix} \quad B = \begin{bmatrix} -1.0 & 0.0 \\ -1.0 & -1.0 \end{bmatrix}.$$

Using the LMI toolbox in MatLab, the maximal admissible time delay for stability are: $h = 4.472$ for $\dot{h}(t) = 0$, $h = 3.604$ for $d = 0.1$, $h = 0.9999$ for $d \geq 1$ according to Theorem 1, and $h = 0.9999$ without knowing anything about $\dot{h}(t)$ according to Corollary 1. For the constant delay case $(\dot{h}(t) = 0)$, the same result is also obtained in [3]. Again, to the best of our knowledge, $h = 4.47$ is the largest bound obtained in the literature for this system with constant delay. However, the conditions in [3] do not hold for $d \geq 1$, and a completely different and considerably sharp derivation is used for our results. For other comparisons, a summary is given in Table I.

Consider another example for (1a) with time delay satisfying case A2)

$$A = \begin{bmatrix} -2 & 0 \\ 1 & -3 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 0 \\ -0.8 & -1 \end{bmatrix}$$
$$\|\Delta A(t)\| \leq 0.2 \quad \|\Delta B(t)\| \leq 0.2.$$

The uncertainties of the aforementioned system are of the forms (2) with

$$D = E = \mathrm{diag}\{\sqrt{0.2}, \sqrt{0.2}\}$$
$$G = H = \mathrm{diag}\{\sqrt{0.2}, \sqrt{0.2}\}.$$

According to Corollary 1, the system is robust and asymptotically stable for all $0 \leq h(t) \leq 0.8522$. For comparisons, see Table II.

The two examples conclusively show that our results are less conservative than the previous ones.

V. CONCLUSION

By using unique Lyapunov–Krasovskii functionals, new stability conditions for a class of linear uncertain systems with a time-varying time-delay are obtained. Effectiveness of the proposed Lyapunov–Krasovskii functionals indicates that a proper distribution of the time delay in the Lyapunov–Krasovskii functionals is crucial to obtain less conservative criteria. Nevertheless, the approach for distributing the delay terms is yet to be developed. Also, our results are more general than some previous ones since they can be used even if the time-derivative of the time-delay is larger than 1.

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