Compound Particle Optimization Using Speciation for Multimodal Function Optimization

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Abstract

Multimodal optimization problems pose a new challenge to evolutionary computation, since they usually not only require a search for one global optimum, but also simultaneously locating multiple optima. This paper presents a new variant of particle swarm optimization, which incorporates the notion of speciation into the compound particle optimization for solving multimodal functions. In the proposed species-based compound particle swarm optimization (SCPSO), several species containing compound particles are adaptively formed according to their similarity at each iteration step. The corresponding techniques of the compound particle, which are inspired by physics mechanisms, provides successive local improvements for each species to precisely and quickly identifying multiple global optima. Experiments on multimodal test functions suggest that SCPSO is more computationally efficient than the conventional species-based PSO.

1. Introduction

Many real-world optimization tasks are subject to multimodal and always require optimization algorithms capable of simultaneously locating multiple global optima, such as in engineering design, economic modeling, and traffic systems [1][3]. Particle swarm optimization (PSO) has been increasingly used as an efficient optimization technique for unimodel functions in a variety of contexts. However, the basic PSO method proposed by Kennedy and Eberhart [9] was primarily designed to search for a single global optimum, and it is expectable to incorporate some approaches to improve the performance of PSO for multimodal optimization functions.

In recent years, several techniques have been applied to address multimodal optimization problems from evolutionary algorithms (EAs) community, such as crowding [14], fitness sharing[6], restricted tournament selection [7], niching [1][3][5][15] and speciation [10]. Similar to EAs, PSO has also been investigated to solve multimodal optimization problems with promising results [2][8], one prominent approach called species-based particle swarm optimization (SPSO) is specially designed for simultaneously locating multiple global optima [12][13]. Within SPSO, different species which may explore potential optima are formed based on the feedback obtained from the fitness landscape.

The compound particles swarm optimization (CPSO) is a newly proposed algorithm inspired by principles from the domain of physics [11]. It focuses on exploring the search space more comprehensively to ensure a good tracking behavior in dynamic environment, where the evaluation function and/or environmental conditions may change over time due to many factors.

In this study, one method incorporates the notion of speciation into CPSO, called SCPSO is proposed for simultaneously locating all optima for multimodal problems. Within the SCPSO, it is hoped that the compound particles may help to enhance the local search ability as well as maintain diversity within each species, experimental studies on a series of benchmark multimodal functions validate the efficiency of SCPSO for solving multimodal optimization problems.

This paper is organized as follows: Section 2 provides a brief introduction of the conventional PSO, the notion of speciation and the recently proposed CPSO. Section 3 describes the SCPSO in details, including construction of compound particles, the special scheme of information communication among member particles. Section 4 provides experimental results and analysis over some multimodal functions. Finally, the conclusions and directions for future research are given in Section 5.

2. Related Research

2.1. Particle Swarm

Particle swarm optimization simulates the social behavior of insects and animals, instead of using evolutionary operators as in EAs, PSO employs autonomous particles to search for optima, each particle adjusts position according to the best position ever found by itself, and by the particles in the entire swarm or neighbors. For each particle, the update of velocity and position in $D$-dimensional search space at each iteration is described as follows:
2.3. Compound Particle Swarm Optimization

Species-seeds are used to locate different optima. It is hoped that species seeds will provide right guideness in the same species. In this way, it will be a new seed. Note that individuals will be attracted by the species seed within the same species, within the radius are identified to belong to the same species, best-fit individual in the species. Those particles that fall within the radius are identified to be potential optima. Seeds (the most dominating individual in each species) can be identified to be potential optima. By applying this method at each iteration, multiple species can be identified to be potential optima. The radius measured in Euclidean distance from the best-fit individual in the species. Those particles that fall within the radius are identified to belong to the same species, by applying this method at each iteration, multiple species seeds (the most dominating individual in each species) can be identified to be potential optima.

The method of determining speciation adopted in [12] are described as below: all individuals are sorted in decreasing order of fitness, and are checked in turn against species seeds identified so far, if an individual does not fall within the radius of all discovered seeds, the individual will be signed to be a new seed. Note that individuals will be attracted by the species seed within the same species. In this way, it is hoped that species seeds will provide right guideness in species to locate different optima.

2.3. Compound Particle Swarm Optimization

In physics domain, the compound particle refers to a kind of special particles, which are composed of different particles through chemical actions, this popular technique allows compound particles holding more predominant characters than properties of member particles. In [11], Liu has suggested a method called compound PSO (CPSO) based on the principles of compound particle in physic domain, to address dynamic optimization problems which require algorithms continuously tracking the moving optima.

In CPSO, the compound particle is created as a simple geometrical structure, a triangle, which composed of three particles named as member particles. The member particles communicate through a self-adjustment operation, which will draw the compound particles explore more comprehensive space and hence good tracking in dynamic environment. Moreover, the identification of representative particle is proposed to maintain swarm diversity as well as guarantee the search precision in the fitness landscape, and a operation called integral movement is employed to preserve information during the optimization process.

3. Species-based Compound Particle Swarm Optimization

Two major considerations should be addressed when solving multimodal problems: searching different promising areas where optima may locate, and exploiting good solutions around the possible areas. In order to precisely and quickly find optima.

As discussed above, it is reasonable to combine the notion of speciation and compound particles to improve PSO’s adaptation in a multimodal fitness landscape.

This paper describes a species-based compound particle optimization (SCPSO) incorporates compound particles together with special mechanisms into each species, it shares the basic framework of the conventional PSO, and the key operations for the compound particle in the proposed algorithm are described in details.

3.1. Constructing Compound Particles within a Species

Within CPSO, after creating several species, compound particles will be constructed by three neighboring individuals in each species. Figure 1 summarizes steps for creating speciation together with the construction of compound particles.

In this way, the compound particle is designed to be composed of three member particles sharing similar features, and it primarily considers to improve the weaker individuals utilizing the synthetical information from the better neighbors. This allows some useful information held by non-dominant ones could be sufficiently developed to help efficiently searching around optima.

3.2. Adopted Schemes for the Compound Particle

The operations for the dynamics of compound particles introduced in [11] is adopted here. There are three main strategies described as below:

1. Velocity-anisotropic reflection scheme: This scheme aims to drive compound particles to explore the promising search space more comprehensively, it works by replacing the worst member particle with another reflection point toward the better direction derived from positions of the other two ones, a velocity-anisotropic reflection vector that has different constituents related to velocities for each dimension is applied to drive the compound particle to fully explore the D-dimensional search space.
The reflection operation is illustrated in Figure 1. The position of the worst particle in a compound particle is denoted as \( W \) and the position of the central point of the other two member particles is denoted as \( C \). Then, the compound particle is reflected in accordance with the point \( W \) to a point \( R \). An expansion will be made in that direction to point \( E \) when the solution of point \( R \) is fitter than that at point \( W \). The reflection point \( R \) and the extension point \( E \) are calculated as follows:

\[
\vec{WR} = \vec{WC} + \vec{\gamma} \times \vec{WC} \tag{4}
\]

\[
\vec{WE} = \theta \times \vec{WR}, \quad \text{if } f(R) > f(W), \tag{5}
\]

where \( \vec{\gamma} \) is the reflection vector and \( \theta \) is the extension factor.

For the purpose of ensuring the VAR vector can drive compound particles to explore in the \( D \)-dimension search space, each constituent in the VAR vector \( \vec{\gamma} \) is generated as follows:

\[
\gamma_{ij} = \text{rand}(0, e^{-|v_{ij}/v_{max}|}), \quad j \in (1, 2, \cdots, D), \tag{6}
\]

where \( v_{ij} \) and \( \gamma_{ij} \) are the velocity and the reflection velocity of the \( i \)-th compound particle in the \( j \)-th dimension respectively.

In this way, the exploration ability will be adaptively adjusted based on the feedback of the fitness landscape.

(2) **Identifying the representative particle:** Different from SPSO, each member particle in SCPSO will be guided by the representative particle in the same compound particle, this may promote diversification within species. Here, two factors including the fitness and Euclidean distance are both take consideration to identify the representative particle based on the following probability:

\[
P_{ci} = \beta p_{ci}^d + (1 - \beta) p_{ci}^f, \tag{7}
\]

where \( P_{ci} \) is the probability that the \( i \)-th member particle of the \( c \)-th compound particle becomes the representative particle, and \( \beta \) is the identification factor. \( p_{ci}^d \) and \( p_{ci}^f \) represents the proportion of the total distance of the \( i \)-th member particle in the \( c \)-th compound particle and the proportion of the fitness respectively.

(3) **Integral movement:** One of the major characters of the compound particle is the physical globality, that is, the "particle" is considered to be a whole to participate in chemical reactions. In SCPSO, the velocity of a representative particle is conveyed to the other two member particles in the compound particle. The aims are to avoid trapping in local optima when moving to a "better" space as well as preserving valuable information for the successive iterations.

**4. Experimental Study**

**4.1. Experimental Design**

In this study, four benchmark multimodal test functions have been adopted to compare the proposed SCPSO with the basic SPSO, these functions are described in Table 1, and Deb’s 1st Function (F4) is relatively simple to solve even though there is little separation between peaks, Brainin RCOS (F1) and Himmelblau (F2) both have peaks with large catchment areas, whereas F2 may cause PSO locating the first found peak but neglecting exploring other global peaks. Six-Hump Camel Back (F4) has 4 local optima to be trapped in for particles.

In this study, two performance measurements proposed in [13] are adopted as below:

(1) **Accuracy:** It measures the closeness between all known global optima and the closest species seed, and is calculated as below:

\[
\text{Accuracy} = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{|f^* - f_{	ext{opt}}|^\alpha}
\]


Table 1. Multimodal Test Functions.

<table>
<thead>
<tr>
<th>Function</th>
<th>Function Name</th>
<th>Range</th>
<th>Optimal $r_{opt}$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>Branin R.Cos: $F_1(x,y) = (y - \frac{x^2}{4} + \frac{\pi}{2} - 6)^2 + 10(1 - \frac{1}{8}\cos(x)) + 10$</td>
<td>$-5 \leq x \leq 10$</td>
<td>4</td>
<td>5 global optima</td>
</tr>
<tr>
<td>F2</td>
<td>Himmelblau: $F_2(x,y) = 200 - (x^2 + y - 11)^2 - (x + y^2 - 7)^2$</td>
<td>$-6 \leq x,y \leq 6$</td>
<td>3</td>
<td>4 global optima</td>
</tr>
<tr>
<td>F3</td>
<td>Six-Hump Camel Back: $F_3(x,y) = -4(4 - 2.1x^2 + \frac{x^4}{4})^2 + xy + (-4 + y^2)y^2$</td>
<td>$-1.9 \leq x \leq 1.9$</td>
<td>1</td>
<td>2 global optima and 4 local optima</td>
</tr>
<tr>
<td>F4</td>
<td>Deb’s 1st Function: $F_4(x) = \sin^6(5\pi x)$</td>
<td>$0 \leq x \leq 1$</td>
<td>0.15</td>
<td>5 equally spaced global optima</td>
</tr>
</tbody>
</table>

accuracy = $\frac{1}{||opts||} \sum_{j=1}^{||opts||} |fit(opt_j) - fit(seed_j)|$  (8)

where $||opts||$ is the number of known global optima. For each optimum $opt_j$, the distance from its corresponding closet seed $seed_j$ is calculated to check the search precision within a certain number iteration steps.

(2) Convergence speed: In this research, a global optimum is considered to be found by checking the dominating individual in a species to see if it is close enough to a known global optimum, it can be measured in terms of the number of evaluations required to achieve a pre-specified accuracy acceptance threshold ($0 < \xi < 1$), and the following equation should be fulfilled:

$$\forall x \in S_{opt} \exists y \in S_{seed}: \min ||x - y|| \wedge ||fit(x) - fit(y)|| \leq \xi$$  (9)

where $S_{seed}$ is the set of identified species seeds, and $S_{opt}$ is a set of all known global optima.

Another measurement is the success rate, it is measured by the percentage of runs in which optima are located successfully within 2000 iteration steps, the accuracy threshold $\xi$ was to 0.0001. All results were averaged over 30 runs.

For PSO parameters, $\phi_1$ and $\phi_2$ were both set to 2.05. The constriction factor $\chi$ was set to 0.729844[4], $V_{max}$ was set to be the lower and upper bounds of the allowed variable ranges.

4.2. Experimental Results and Analysis

(1) Accuracy: Table 2 shows the results of accuracy for SPSO and SCPSO on all functions. The population size was set to 50, and the identification factor $\beta = 0.5$, implying the ingredients of fitness and distance have equal strength.

As can be seen in Table 2, both SPSO and SCPSO have gained 100% success rate, which indicates that the speciation method could efficiently finding all optima within the required accuracy $\xi$ of 0.0001 within 2000 iterations in the investigated test problems. In addition, SCPSO produced a better accuracy than SPSO for all four test functions, this indicates that the interaction mechanism of velocity-anisotropic reflection is helpful for a fined search around promising areas.

(2) Convergence speed: In this part, parameter setups were applied as in the above subsection, Table 3 compares the convergence speed of SPSO and SCPSO on all listed functions. These results show that SPSO with the compound particles is more effective in deriving better locating performance in multimodal optimization problems than using speciation alone.

(3) Sensitivity to population size: The swarm size has an important influence on the performance of PSO to solve problems. Figure 5 shows the results of SCPSO with the swarm size set to different value in the range of [10,100].

From Figure 5, it can be seen that a better result can be obtained when the population size is in the range of [30,60] on all functions, that is, adding redundant particles could not provide help for performance improvement, and in some extent, this also give evidence that SCPSO could maintain sufficient diversity for handling the investigated multimodal problems. With advantages of precisely identifying each promising areas and fully exploring these areas, SCPSO could efficiently locating all global optima in the search space.

Table 2. Results on accuracy after 2000 iterations with 100% success rate(mean and standard deviation).

<table>
<thead>
<tr>
<th>Function</th>
<th>SPSO</th>
<th>SCPSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>1.98E-6±2.45E-7</td>
<td>0.00E+0±0.00E+0</td>
</tr>
<tr>
<td>F2</td>
<td>9.78E-1±1.65E+0</td>
<td>7.25E-2±1.51E+0</td>
</tr>
<tr>
<td>F3</td>
<td>4.25E-3±4.45E-13</td>
<td>3.18E-6±9.25E-15</td>
</tr>
<tr>
<td>F4</td>
<td>1.86E-5±7.25E-9</td>
<td>3.25E-7±8.15E-15</td>
</tr>
</tbody>
</table>

Table 3. Number of evaluations required to find all optima(mean and standard deviation).

<table>
<thead>
<tr>
<th>Function</th>
<th>SPSO</th>
<th>SCPSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>4083±1278</td>
<td>3842±515</td>
</tr>
<tr>
<td>F2</td>
<td>5370±1428</td>
<td>4285±1272</td>
</tr>
<tr>
<td>F3</td>
<td>4215±1023</td>
<td>3915±925</td>
</tr>
<tr>
<td>F4</td>
<td>3021±927</td>
<td>2857±789</td>
</tr>
</tbody>
</table>
5. Conclusion

This paper proposes an extension to the compound PSO by using the notion of speciation to address multimodal optimization problems. The proposed SCPSO applies the species identification technique to help particles searching for possible areas where multiple global optima may locate, and incorporates mechanisms of compound particles to further improve its local search ability and hence a good locating behavior. By using the velocity-anisotropic reflection scheme, the fitness space is fully explored and contribute to a comprehensive search in the multi-dimensional search space. By selecting the representative particles and the integral movement, the information held by particles, no regardless of superior or inferior, will be utilized for a rigorous searching for multiple peaks.

SCPSO proves to be an efficient algorithm for the investigated multimodal optimization problems in this study. For the future work, SCPSO will be applied to more complex real-world optimization problems, including in dynamic contexts and high-dimensional functions, which have more challenging to address using traditional technique. Furthermore, it is valuable to make a deep investigation on comparing the performance of SCPSO with other state-of-art methods in the multimodal fitness landscapes.

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References


