ABSTRACT
This paper addresses on the robust stability problem of interval polynomials and matrices of the continuous-time linear system (C-TLS) and discrete-time linear system (D-TLS) that contain the real uncertain parameters. The robust stability of the interval polynomials and matrices can be determined by three different robust stability criterions that check the global minimum of each order Hurwitz determinant or check the global minimum of each element in first column of Routh array. The linear matrix inequality (LMI) methods and parameters dependent Lyapunov functions (PDLY) methods are often used to effectively determine the robust stability of the interval matrices and polynomials, and in this paper, these robust criterions are also effective to determine the stability of the interval matrices and polynomials. The robust stability of the interval matrices can be transformed into the stability of an interval polynomial too. The third robust criterion can reduce the number of the optimization objectives, and as well as the computational complexity when determining the robust stability of the interval polynomials and matrices. Through applying the bilinear transformation, the three robust criterions can be extended into the interval polynomials and matrices of discrete-time linear system. Different examples of interval polynomials and matrices are studied to show the effectiveness and accurateness of the robust stability checking methods.

KEY WORDS
Robust stability, real uncertain parameters, interval polynomials and matrices, bilinear transformation, robust stability criterions

1. Introduction
The robust stability problems of the interval polynomials and matrices have been studied considerable over the past decades years. Kharitonov’s seminal robust stability criterion on the interval polytope polynomials and matrices in [16] are considered as the breakthroughs in the stability of the interval polynomials, because the stability of interval polynomials and matrices can be checked by four Kharitonov-defined vertex polynomials. A note on this theorem is presented in [18], however, a counterexample of the interval polynomial and matrix are proposed by the authors in [1], the counterexample demonstrates that: the Kharitonov theorem may fail to determine the robust stability of a class of interval polynomials whose coefficients are fixed by the multiple uncertain parameters’ the continuous mapping, and the corresponding comments on the counterexample are given in [7]. Based on the parameter dependent Lyapunov functions methods, the authors in [2] propose a robust stability criterion of the interval linear systems, the robust stability criterion in [2] determines the robust stability of the interval polynomials and matrices through checking the Hermite matrix whether it belongs to the positive definite matrix (PDM) for arbitrary uncertain real parameters in the prescribed set, where the Hermite matrix in [2] is a polynomial parameter dependent Lyapunov interval matrix. Motivated by the Kharitonov’s theorem, the authors in [3] propose and point out that an interval polynomial with uncertain real parameters lying in a diamond is a Hurwitz polynomial if and only if the eight distinguished extreme polynomials are the Hurwitz stable polynomial. In [4], a linear matrix inequality (LMI) method is proposed to check the robust stability of the implicit polynomials through a polynomial Lyapunov function. In [5], the authors apply the small gain analysis, circle analysis, positive real analysis, and Popov analysis to determine the maximum perturbation bound of the uncertain parameters when the system keeps the robust stability, thus the interval scope of system parameters can be obtained, and then the analysis can indirectly determine the robust stability of the interval polynomials as well. In [6], the authors present a framework of the parameter dependent Lyapunov functions and the Popov analysis for the robust stability of the interval matrices. In [8-10], the sufficient robust stability conditions in the LMI form are proposed for the uncertain continuous-time linear system, and parameter dependent Lyapunov functions are used to assure the robust stability of the uncertain linear system. A test is proposed for determining and analyzing the robust stability or performances of the uncertain linear system in [11], where the test is the extension of the quadratic stability’s notions. A sufficient condition of the Hurwitz stability via parameter dependent Lyapunov function is presented for the interval matrices in [12]. Based on parameter dependent Lyapunov function approach, the conditions in term of the LMI for the robust stability analysis and robust H2 performance of the uncertain interval linear system are presented in [13] and [14] respectively. In [15], stability analysis of interval polytope uncertain linear system are presented in term of two LMI conditions based on the quadratic stability and parameter dependent Lyapunov functions, where the actuators of the control system are subjected to amplitude saturation. Necessary
and sufficient conditions for component-wise stability of the interval matrices are presented in [19]. In [20], the author develops the necessary and sufficient conditions for types of stability, complete instability, and composite stability for a special class of interval matrices. The robust stability of discrete-time difference equations is studied in [21], where the author presents methods to determine the maximum allowable perturbation of interval polynomials when the system preserves the Schur stability. In [22], the author provides a simple sufficient condition for the Hurwitz stability of the convex combination of two real matrices. In [35-36], the authors provide an algebraic criterion with sufficiency to check the robust stability of the interval polynomials or matrices. The methods of determining the robust stability of the uncertain linear systems can be roughly divided into following cases: the Kharitonov’s theorem and edge theorem methods, interval coefficients numerical algebraic methods, parameter dependent Lyapunov function methods, quadratic stability theorem methods, and LMI methods. The Kharitonov’s theorem method is simple and effective to check the robust stability of the interval polynomials or matrices when assessing a class of special interval polynomials and matrices, but sometimes this method may fail to assess the robust stability of the interval polynomials or matrices that contain multiple uncertain real parameters, and the corresponding counterexample is given in [1]. The quadratic stability of the interval polynomials and matrices are usually conservative than the actual robust stability. The quadratic stability can infer the robust stability of the interval system, but the converse is not true. The results of quadratic stability are usual in term of the LMI forms. The parameter dependent Lyapunov function and LMI method are often applied to check the robust stability of the interval system as well, and the conservatism of PDLF and LMI method is less than the quadratic stability method [38]. In this paper, without using the LMI methods, PDLF methods, and quadratic stability methods, we apply three robust stability criterions to check the robust stability of the interval polynomials and matrices. The paper is arranged as follows: the Section.2 introduces the problem statements about checking the linear system’s stability of the interval polynomials and matrices; the Section.3 presents the robust stability criterions and assesses the robust stability of the examples of interval polynomials and matrices; in Section.4, the robust stability criterions are extended into the interval matrices and polynomials of the discrete-time linear system; the Section.5 briefly reviews and concludes the paper.

2. Problem statements

The model of the controlled process is a mathematical description of the physical system, and the model depends on making various approximations of the physical systems, therefore the model have inaccuracies and deviations. We assume the uncertainties and model inaccuracies of the linear systems and linear controlled plants can attribute to the uncertain parameters, in this paper. Uncertainties may lead to perturbations in the coefficients of the system’s characteristic polynomial, which consequently may jeopardize the stability of the control system. Perturbations in coefficients of the polynomial may represent the working point and condition variations of the processes or plants. Hence, in this paper we focus on the problem of checking the robust stability of the interval polynomials and matrices, but the uncertain parameters of the interval matrices and polynomials are only known within the given bounds. So, the robust stability problem of the interval polynomials and matrices focus on whether the stability of interval polynomials and matrices are preserved in the all given bounds of the uncertain parameters. Let us consider the polynomial equation and state equation of continuous time linear system

\[ P(s,a) = s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n \]

Where \( a_i \) is the uncertain coefficient of the interval polynomial, \( a_i^l \) and \( a_i^u \) are the lower bound and upper bound of the uncertain parameters, \( q \) is a vector of uncertain parameters in the given set \( Q \). The state matrix \( A(q) \) depends on uncertain parameter vector \( q \). In this paper, we define the Hurwitz polynomial and Hurwitz matrix that should satisfy all the solutions of this interval polynomial \( P(s,a) \) equation and all the eigenvalues of the interval matrix \( A(q) \) should have negative real part, then the continuous-time linear system's interval polynomial \( P(s,a) \) and interval matrix \( A(q) \) belong to the Hurwitz polynomial and Hurwitz matrix respectively. The problem of checking the robust stability of the interval matrix \( A(q) \) can be transformed into the robust stability of the interval polynomial. The robust stability of the interval matrix \( A(q) \) can be defined as follow:

\[ R(eig\{A(q)\}) \leq 0, \forall q \in Q \Rightarrow \forall q \in Q, P(s,q) = det(sI - A(q)) = H(s) \]

Where \( H(s) \) is the set of all the Hurwitz stable polynomials of the continuous-time linear system, \( eig\{\} \) is the eigenvalue operator of the matrix. Therefore, the robust stability of the interval matrix \( A(q) \) is ultimately transformed into the robust stability of the interval polynomial. Once the robust stability problem of the interval polynomial is settled, and then the corresponding robust stability problem of the interval matrix is solved as well.

3. Assessing robust stability of polynomials and matrices

In this paper, we assume that the uncertain coefficients of interval polynomials are only known within the given bounds or prescribed set. So, the robust stability problems focus on whether the interval polynomials or matrices are robust stability when all the uncertain parameters perturb in the given bounds.

Theorem.1:

\( P(s,a) \) is an interval polynomial that represent the uncertainties of the model of continuous-time linear system. Un-losing the generality, the interval polynomial
The interval polynomial $P(s,a)$ of the continuous-time linear system has the following form

$$P(s,a) = s^r + \sum_{i=0}^{r} a_i s^{r-i} \in R^r$$

Where $R^r$ is the positive real set, and $a_i$ is the positive real bounded uncertain coefficients of the $P(s,a)$, and $a$ is the uncertain coefficient vector. Thus, sufficient and necessary conditions of the interval polynomial $P(s,a)$ that belong to Hurwitz stable polynomial if and only if:

$$P(s,a) = s^r + \sum_{i=0}^{r} a_i s^{r-i} \in H(s) \Rightarrow
\begin{align*}
c_i^j &= \min \{c_i^{(j)}(a)\} > 0, (j = 1,2,...,n+1) \\
&\text{or: } c_i^j = 0, c_i^j \cap R^- = \emptyset, (j = 1,2,...,n+1) \\
&\text{or: } c = [c_1^1, c_1^2, ..., c_1^n, c_2^1, ..., c_2^n, ..., c_n^1, 0 \leq c', c' \cap R^- = \emptyset]
\end{align*}$$

Where $c_{i,j}$ is the $j$-th column element of the Routh array’s first column, and $c_{1,j}$ is the global minimum of the $c_{1,j}$, and $c'$ is the set of the $c_{1,j}$ ($j=1,2,...,n+1$), $a$ is the uncertain coefficients vector, $R^r$ is the negative real set. The global minimum of each element $c_{1,j}$ of Routh array’s first column should be a positive real, or rather, each term $c_{1,j}$ of Routh array’s first column of the interval polynomial $P(s,a)$ should have a positive real minimum in the given bound. In this paper, the minimum refers to the absolute or global minimum, and we call the first column of the Routh array as the Routh column.

**Proofs of theorem.1**

**Sufficient:**

The interval polynomial $P(s,a)$

$$P(s,a) = s^r + \sum_{i=0}^{r} a_i s^{r-i} \in H(s)$$

We define the set of all the Hurwitz stable polynomials $H(s)$

$$H(s) = \{p(s) : P(s) \in R^r, \Re(P(s) = 0) < 0\}$$

Where $P(s)$ is a constant polynomial, and the element $c_{1,j}(a)$ is the $j$-th column element of the Routh column of the interval polynomial $P(s,a)$, the element $c_{1,j}(a)$ has a general formula

$$c_{1,j}(a) = F_j(a_0, a_1, ... , a_j), (j = 1,2,...,n+1)$$

The following relationships are satisfied

$$c_i^j = \min \{c_i^{(j)}(a)\} = \min \{F_j(a_0, a_1, ... , a_j)\}, (j = 1,2,...,n+1)$$

Then there exists $a_{i+}$ satisfying the equation

$$3a_i \in [a_i, a_{i+}], c_{i+} = \min \{F_j(a_0, a_1, ... , a_j)\} = F_j(a_{i+}, a_{i+}, ... , a_{i+})$$

For $\forall a_i \in [a_i, a_{i+}]$, then we have following relationships

$$c_i^j = F_j(a_{i+}, a_{i+}, ... , a_{i+}) \geq c_i^j = F_j(a_{i+}, a_{i+}, ... , a_{i+}) - \min \{F_j(a_0, a_1, ... , a_j)\} > 0$$

Thus, the following inequalities can be obtained

$$c_i^j = F_j(a_{i+}, a_{i+}, ... , a_{i+}) > 0, (j = 1,2,...,n+1)$$

Known from the Routh-Hurwitz Criterion (RHC) in [25-27], hence the polynomial $P_0(s,a_0)$ is a Hurwitz stability polynomial

$$\forall a_0 \in [a_i, a_{i+}], P_0(s,a_0) = s^r + \sum_{i=0}^{r} a_i s^{r-i} \in H(s)$$

The interval polynomial $P(s,a)$ is a Hurwitz stable polynomial

$$\forall a_i \in [a_i, a_{i+}] \in R^r, P(s,a) \in H(s)$$

**Necessary:**

Because the interval polynomial $P(s,a)$ belongs to the Hurwitz stable polynomial

$$P(s,a) = s^r + \sum_{i=0}^{r} a_i s^{r-i} \in H(s), a_i \in [a_i, a_{i+}] \in R^r$$

We know from the RHC, and the following inequalities of the Routh column should satisfy the relationships

$$c_{i,j}^j = F_j(a_{i+}, a_{i+}, ... , a_{i+}) > 0, (\forall j = 1,2,...,n+1)$$

Thus, obviously the inequalities of Routh array’s first column satisfy the relationships

$$c_{i,j}^j = \min \{c_{i,j}(a)\} > 0, (\forall j = 1,2,...,n+1)$$

Hence, the theorem.1 satisfies, and the proofs end.

**Remark.1:**

The stability of continuous-time linear invariant systems can be directly checked by the Hurwitz determinant criterion (HDC) and Routh array criterion (RAC). The detail contents of the Routh stability criterion and Hurwitz stability criterion, and also their relationships can refer to the references [24-27]. The robust stability of the interval polynomials or matrices can not be directly checked by HDC and RAC. When we check the interval polynomial whether it belongs to the Hurwitz stable polynomial, first, we should check the necessary condition of uncertain coefficients. We know that the uncertain coefficients $a_i$ is the subinterval of positive real set $R^r$. If the uncertain coefficient $a_i$ of the interval polynomial contains any non-positive real, in other words, any uncertain coefficients of the interval polynomial include zero or negative real, and then we can assert that the uncertain coefficients must not belong to Hurwitz stable polynomial. Thus, all the uncertain coefficients should belong to the positive real or positive real interval in the prescribed set, which is the uncertain coefficients’ necessary condition of the Hurwitz stable interval polynomial. The coefficient necessary condition of interval polynomial that belongs to the Hurwitz stable polynomial is $a_i \in [a_{i-}, a_{i+}] \in R^r$. When the uncertain coefficients of the interval polynomial satisfy the necessary conditions, after that we can use the theorem.1 to check the robust stability of interval polynomials. However, the uncertain coefficients of the interval polynomials $P(s,a)$ can satisfy the necessary conditions in most cases, checking the theorem.1 is the core task of determining the interval polynomial whether it belongs to Hurwitz stable polynomial. If the minimum $c_{i,j}^j = \min \{c_{i,j}(a)\}$ ($j=1,2,...,n+1$) is a non-positive real, then we can conclude that the interval polynomial $P(s,a)$ is not a Hurwitz stable polynomial. As we known, the Routh stability criterion and Hurwitz stability criterion are equally to check the stability of the continuous linear time-invariant control system (CLTICS). The general form of the Hurwitz determinant $H_a(a)$ in the Appendix.1, and the Routh column has relationships with each Hurwitz determinant, and the elements of Routh column can be expressed in terms of the Hurwitz determinant as follows [24-27]:

$$c_{i,1} = a_0, c_{i,2} = a_1, c_{i,3} = H_2(a) / H_1(a), c_{i,4} = H_3(a) / H_2(a), ..., c_{i,n} = H_{n+1}(a) / H_{n}(a)$$

The Fig.1 in Appendix.1 shows the calculating process of the Routh column for the continuous linear time-invariant
polynomials or interval polynomials that can represent the characteristic equation of the closed-loop control systems. We can use the recursive formula in [27] to computing the Routh column element $c_{ij}$ (j=1,2,…,n+1) for the CLTICS or interval polynomials.

$$\beta = a_i - \frac{a_j}{a_i} \beta, \quad a_i = 1, \quad a_j = a_i,$$

$$a_j = a_i, \quad (i = 1, 2, …, n-1, \quad j = 2, 4, 6, …, \text{even}) \quad (6)$$

**Inference.1:**

$P(s,q)$ is a class of special interval polynomial since the uncertain coefficients of the interval polynomial depend on the multi-affine continuous mapping of uncertain parameters. The polynomial $P(s,q)$ has following form

$$P(s,q) = s^n + \sum_{i=1}^{n} a_i s^{n-i} \in H(s) \Leftrightarrow c_j = \min\{c_j(q)\}_{0 \leq i \leq n+1} \quad (8)$$

Where $q_j$ is the real bounded scalar uncertain parameters, $s$ is the real set, $R^\ast$ is the positive real set, and $a_i$ is the positive real bounded coefficients of interval polynomial that depends on the multi-affine continuous mapping of the uncertain parameters $q_i$. The sufficient and necessary conditions of interval polynomial $P(s,q)$ that belongs to Hurwitz stable polynomial if and only if:

$$P(s,q) = s^n + \sum_{j=1}^{n} a_j s^{n-j} = s^n + \sum_{j=1}^{n} a_j s^{n-j}, \quad a_j = f_j(q_1,q_2,\ldots,q_j) \in R, \quad 0 < M_1 \leq a_i \leq M_j, \quad a_j = f_j(q_1,q_2,\ldots,q_j) \subset R, \quad (i = 1, 2, …, n-1; \quad j = 2, 4, 6, …, \text{even}) \quad (7)$$

**Remark.2:** the inference.1 is an extension of the theorem.1, the coefficients of the interval polynomial depends on the multi-affine continuous mapping of uncertain parameters $q_i$. The coefficients of interval polynomial can be the linear function or nonlinear function of the uncertain parameters. In this case, the Kharitonov’s theorem and its edge theorem maybe fail to check the stability of this class of interval polynomials. A counterexample has been presented by the authors B.ross Barmish, M.Fu, and S.Saleh in paper [1], but the presented method can check the stability of this class of the interval polynomials $P(s,q)$, which is shown in the section.3. The inference.1 can be used to determine the robust stability of the complex interval polynomial.

**Theorem.2:**

Let us consider the two types of the interval polynomials in the theorem.1 and inference.1 respectively, and the interval polynomial $P(s,a)$ and $P(s,q)$ have the following forms:

**case 1: $P(s,a) = s^n + \sum_{i=1}^{n} a_i s^{n-i} \in R^\ast; (i = 1, 2, …, n)$**

**case 2: $P(s,q) = s^n + \sum_{j=1}^{n} f_j(q_1,q_2,\ldots,q_j) s^{n-j} = s^n + \sum_{j=1}^{n} a_j s^{n-j}, \quad a_j = f_j(q_1,q_2,\ldots,q_j) \subset R, \quad 0 < M_1 \leq a_i \leq M_j; (i = 1, 2, …, n; \quad k = 1, 2, …, m) \quad (9)$$

The sufficient and necessary conditions of the interval polynomial $P(s,a)$ and $P(s,q)$ that belong to the Hurwitz stable polynomial if and only if [39]:

$$\text{case 1}: P(s,a) = H(s) \Leftrightarrow H'_j = \min\{H'_j(a)\} \subset R^\ast; (i = 1, 2, …, n)$$

<table>
<thead>
<tr>
<th>Case</th>
<th>Condition</th>
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<tbody>
<tr>
<td>1</td>
<td>$H'_j = \min{H'_j(a)} &gt; 0, (\forall j = 1, 2, …, n)$</td>
</tr>
<tr>
<td>2</td>
<td>$H'_j = \min{H'_j(a)} &gt; 0, (\forall j = 1, 2, …, n)$</td>
</tr>
<tr>
<td>3</td>
<td>$H'_j = \min{H'_j(a)} &gt; 0, (\forall j = 1, 2, …, n)$</td>
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<td>4</td>
<td>$H'_j = \min{H'_j(a)} &gt; 0, (\forall j = 1, 2, …, n)$</td>
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$$\text{Case 2}: P(s,q) = H(s) \Leftrightarrow H'_j = \min\{H'_j(q)\} > 0, (\forall j = 1, 2, …, n)$$

**Remark.3:** Each uncertain coefficient $a_i$ of the interval polynomial is the positive real or the subinterval of positive set $R^\ast$, which is the coefficient necessary condition of interval polynomial that belongs to Hurwitz stable polynomial. The sufficient condition necessary condition of interval polynomial that belongs to the Hurwitz stable polynomial is $a_i \in [a_i, a_i] \subset R$ in case.1. If the uncertain coefficient necessary condition can not be certified, then we can assert that the interval polynomial must not belong to Hurwitz stable polynomial, however this coefficient necessary conditions usually can be guaranteed in most cases. Therefore, checking the theorem.2 is the core task of determining an interval polynomial whether it appertains to Hurwitz stable polynomial. If each Hurwitz determinant’s minimum $H'_j \geq \min\{H'_j(a)\}$ and $H'_j \geq \min\{H'_j(q)\}$ (j=1,2,..,n) are nonpositive real in case.1-2, then we can conclude that the interval polynomial $P(s,a)$ and $P(s,q)$ must not belong to Hurwitz stable polynomial. The theorem.1 and theorem.2 are the equivalent theorems, and they both can be used to check the stability of interval polynomials. However, there are some differences between the theorem.1 and theorem.2. To theorem.1, when all the uncertain coefficients of interval polynomial satisfy the coefficients necessary conditions, then the terms $c_{1j}=1$, $c_{1j}=0$, and $c_{j,n}=a_{jn}$ can naturally satisfy the condition $\min\{\} \geq 0$, so the number of the optimization problem is $n-2$. As well as, to theorem.2, when the coefficient necessary conditions are satisfied, then the term $H'_j=0$, naturally satisfies the condition $\min\{H'_j\} \geq 0$, so the number of optimization problem that needs to be optimized is $n-1$. In addition to, the element $c_{ij}$ (j=3,4,…,n) of the Routh column is the fraction function (FF) of the uncertain coefficients in the theorem.1, so the optimization problems in theorem.1 is fraction optimization problems. Whereas each Hurwitz determinant $H_j$ (j=1,2,…,n) is not fraction function in the theorem.2.

**Theorem.3:**

Let us consider the two types of the interval polynomials in the theorem.1 and inference.1 again, the interval polynomial $P(s,a)$ and $P(s,q)$ have the following forms:

$$\text{case 1}: P(s,a) = s^n + \sum_{i=1}^{n} a_i s^{n-i} \in R^\ast; (i = 1, 2, …, n)$$

$$\text{case 2}: P(s,q) = s^n + \sum_{j=1}^{n} f_j(q_1,q_2,\ldots,q_j) s^{n-j} = s^n + \sum_{j=1}^{n} a_j s^{n-j}, \quad a_j = f_j(q_1,q_2,\ldots,q_j) \subset R, \quad 0 < M_1 \leq a_i \leq M_j; (i = 1, 2, …, n; \quad k = 1, 2, …, m) \quad (11)$$

Where $H'(a)$ and $H'(q)$ are the j-th order Hurwitz determinant of interval polynomial cluster $P(s,a)$ and $P(s,q)$ respectively, $q$ and $a$ are the uncertain coefficients vector, $H'_j$ is the minimum of the $H'(a)$ and $H'(q)$, and $H'$ is the set of the $H'(j=1,2,..,n)$
We define the four sequences $Q_1, Q_2, Q_3$, and $Q_4$ in above two cases respectively, the specific $Q_1, Q_2, Q_3$, and $Q_4$ are given in Appendix.1. The sufficient and necessary conditions of the interval polynomial $P(s,a)$ and $P(s,q)$ that belong to the Hurwitz stable polynomial if and only if are that: any one condition of the following four conditions is satisfied (case.1 and case.2).

$$
Q_1 > 0, Q_2 > 0; o_r : 0 \notin \{Q_1, Q_2\}; \{Q_1, Q_2\} \cap R = \emptyset; (1)
$$

$$
Q_1 > 0, Q_2 > 0; o_r : 0 \notin \{Q_1, Q_2\}; \{Q_1, Q_2\} \cap R = \emptyset; (2)
$$

$$
Q_1 > 0, Q_2 > 0; o_r : 0 \notin \{Q_1, Q_2\}; \{Q_1, Q_2\} \cap R = \emptyset; (3)
$$

$$
Q_1 > 0, Q_2 > 0; o_r : 0 \notin \{Q_1, Q_2\}; \{Q_1, Q_2\} \cap R = \emptyset; (4)
$$

The sequence $Q_i > 0$ (i=1, 2, 3, 4) means that all the numbers of $Q_i$ should be positive real in the given set of uncertain parameters. All the elements of one of $Q_1, Q_2$, and one of $Q_3, Q_4$ should be positive, in other words, any one condition of above-four conditions should be satisfied, then the interval polynomial $P(s,a)$ and $P(s,q)$ in case.1 and case.2 are the Hurwitz stable polynomial.

**Remark.4:** The stability of linear time-invariant system can be both verified by Hurwitz determinant criterion (HDC) and Routh array criterion (RAC), as well as, the stability of linear time-invariant system can be determined by the Lienard-Chipart criterion (LCC) too, and the detail contents and proofs of the LCC can refer to the references [28-30]. The sufficient and necessary conditions for the stability of linear time-invariant system are simplified by applying Lienard-Chipart criterion, since the number of computed the Hurwitz determinant is $n/2$ or $(n-1)/2$ when order $n$ is the even or odd, so the corresponding computational of determining the stability of the linear time-invariant system can be reduced too. Through modifying the LCC, the LCC can be used to determine the stability of the interval polynomials. To the theorem.1, inference.1 and theorem.2, the number of the optimization problem that needs to be solved is $n-2$ and $n-1$ respectively, whereas the number of the optimization problem that needs to be solved in theorem.3 is $n/2$ or $(n-1)/2$ when order $n$ is the even or odd. Thus, the number of the optimization problems in theorem.3 is reduced, and the computation is reduced as well. Finally, we need less computation to check out robust stability of the interval polynomial when applying the theorem.3. In the theorem.3, $Q_1 > 0$ or $Q_2 > 0$ is the coefficient necessary condition of the interval polynomial that belongs to Hurwitz stable polynomial, and this necessary condition can be satisfied in most cases. When the coefficient necessary condition is certified, we only need to check the sequence $Q_1 > 0$ or $Q_2 > 0$, then the robust stability of the interval polynomials can be determined. For these robust stability criterions, we need to check the Hurwitz determinant $H_n$ whether it satisfies condition $\min\{H_n\} > 0$ in theorem.2, which usually requires analyzing the most complicated formulations. As well as, checking the last second element $c_{i,0}$ in the Routh column satisfies $\min\{c_{i,0}\} > 0$ in theorem.1, which also needs to analyze the most complicated formulations as well. However, we can avoid analyzing the most complicated formulations via using the theorem.3 to determine stability of interval polynomials. Motivated by the Routh array criterion, Hurwitz determinant criterion, and Lienard-Chipart criterion, then we modify these stable criterions for checking the interval polynomials and matrices in this paper. The theorem.1, theorem.2, and theorem.3 in this paper, we can call them as the Routh robust stability criterion, Hurwitz robust stability criterion, and Lienard-Chipart robust stability criterion respectively. The authors in [2] employ the software Gloptipoly to solve the global optimization problems, in this paper, we use the software Lingo that integrates the interior point, quadratic recognition, linear programming, and branch-and-bound algorithm to certify the global optimality and extract globally optimal solutions for solving the robust stability analysis problems. Some software such as LOQO, Lindo/ Lingo, Lindo API, Maple global optimization toolbox, Matlab optimization toolbox, Matlab genetic algorithm and direct search toolbox, and Matlab freeware Gloptipoly provide different algorithms for solving the optimization problems.

### 3.1 Numerical study examples of continuous-time linear systems

**Example.1**

The interval polynomial $P(s,q)$ of the continuous-time linear system is proposed in [1] and also studied in [2] too. The coefficients of the $P(s,q)$ are determined by two uncertain parameters $q_1, q_2$. The feasible set $Q$ is the bounded closed set on the $R^2$. The interval polynomial is a counterexample of the four stability Kharitonov-defined vertex polynomials. The paper [1] has pointed out that: the four Kharitonov-defined vertex polynomials and the four edges interval polynomial of the $P(s,q)$ are the Hurwitz stability polynomial, but the interval polynomial $P(s,q)$ is a counterexample of the Kharitonov theorem, so $P(s,q)$ is not a Hurwitz polynomial. The counterexample shows that the Kharitonov theorem may fail to determine the robust stability of a class of interval polynomials whose coefficients are fixed by the multiple uncertain parameters’ the continuous mapping. The paper [2] uses parameter dependent Lyapunov functions method to determine the $P(s,q)$ does not belong to the Hurwitz stability polynomial via software Gloptipoly, in this paper, we use three methods to determine the interval polynomial $P(s,q)$ that does not belong to the Hurwitz stable polynomial

$$
P(s,q) \equiv s^4 + (2.56 + q_1 + q_2)s^3 + (2.871 + 2.06 q_1 + 1.56 q_2 + q_1 q_2)s^2 + (3.164 + 4.841 q_1 + 1.56 q_2 + 1.06 q_1 q_2)s + (1.853 + 3.773 q_1 + 1.985 q_2 + 4.032 q_1 q_2)
$$

$$
q_1 \in [0,1], q_2 \in [0, 3], \Omega = [0, 1]\times [0, 3]
$$

For the notational simplicity, the interval polynomial $P(s,q)$ can be rewritten as

$$
P(s,q) = s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = s^4 + \sum_{i=0}^{4} a_i s^{i-4}
$$

$$
a_1 = (2.56 + q_1 + q_2); a_2 = (2.871 + 2.06 q_1 + 1.56 q_2 + q_1 q_2)
$$

$$
a_3 = (3.164 + 4.841 q_1 + 1.56 q_2 + 1.06 q_1 q_2); q_1 \in [0,1], q_2 \in [0, 3]
$$

$$
a_4 = (1.853 + 3.773 q_1 + 1.985 q_2 + 4.032 q_1 q_2); \Omega = [0,1]\times [0, 3]
$$

Where $a_i$ (i=1, 2, 3, 4) is the coefficient of the interval polynomial and function of the uncertain parameters.
Hence, the corresponding each Hurwitz determinant of the interval polynomial \( P(s,q) \) can be obtained
\[
H_i(q) = a_i; H(q) = a_i - a_i.
\]
As well as, the elements of the Routh column are acquired
\[
c_i(q) = (a_i - a_i) / a_i
\]
All the coefficients of the interval polynomial \( P(s,q) \) are the positive real, thus \( Q_1 = 0 \) and \( Q_2 = 0 \) naturally hold, based on the robust stability criterion.

Example 2
Considering a fourth-order interval matrix \( A(q) \) of the continuous-time linear system. Motivated by the examples in [34], the elements of sub-matrix \( A_{11} \) and \( A_{22} \) contain nonlinear functions. Checking the stability of the interval matrix \( A(q) \) may have some difficulties because the interval matrix’s element contains the nonlinear function with four uncertain parameters.

\[
A(q) = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix},
\]

The corresponding interval polynomials in case.1 and case.2 can be got respectively.

case.1: \( P(s,q) = \det(sI - A(q)) = s^4 + (k_1 + k_2 + k_3 + k_4)s^3 + \ldots \).

case.2: \( P(s,q) = \det(sI - A(q)) = s^4 + (k_1 + k_2 + k_3 + k_4)s^3 + \ldots \).

It is easily to validate that all the coefficients \( a_i \) of the interval polynomial \( P(s,q) \) are the positive real in the prescribed set \( \Omega \), hence the coefficients of interval polynomial satisfy the robust stability’s necessary conditions. Let us consider the interval matrix and its polynomial in case.1 and case.2. Based on robust stability criterion.1, the minimum of each Hurwitz determinant \( H_i(q) \) of the interval polynomial \( P(s,q) \) does not satisfy the robust stability criterion.

\[
Q_1 = \min \{H_i(q)\} \geq 0 \quad (i = 1, 2, 3, 4)
\]

The minimum of each Hurwitz determinant \( H_i(q) \) of the interval polynomial \( P(s,q) \) does not satisfy the robust stability criterion. Therefore, we can make a conclusion that the interval polynomial \( P(s,q) \) is not a Hurwitz stable polynomial and does not have the robust stability in the given set \( \Omega \). The robust stability criterion.1 also can be employed to determine the robust stability of the interval polynomial \( P(s,q) \). Likewise, the minimum of the each element of Routh column is acquired in the set \( \Omega \),

\[
c_{11} = \min \{c_{1,i}\} \geq 0, \quad (i = 1,2,3,4,5)
\]

Because \( c_{1,4} \) is negative real. Hence, the interval polynomial \( P(s,q) \) is not a Hurwitz stable polynomial and does not have the robust stability in the given set. To reduce the number of the optimization objectives and computational, we can apply the robust stability criterion.3. Because all the coefficients of the interval polynomial are the positive real, the stability of the interval polynomial can be determined by the sequence \( Q_1 \) or \( Q_2 \). We can obtain the sequence \( Q_1 \) or \( Q_2 \) respectively. \( Q_1 = \min \{H_1(q)\}, \min \{H_2(q)\}\) = \( \{2.56000, -0.77581\} \), and \( Q_2 = \min \{H_3(q)\}, \min \{H_4(q)\}\) = \( \{4.18576, -6.44498\} \) does not satisfy the condition of \( Q_1 \geq 0 \) or \( Q_2 \geq 0 \), thus the interval polynomial is not a Hurwitz stable polynomial and does not have robust stability in the prescribed set \( \Omega \). The robust stability criterion 1-3 can determine the robust stability of the interval polynomials \( P(s,q) \), even though this interval polynomial depends on the multiple uncertain parameters continuous mapping. Therefore, the stability criterion 1-3 can test the robust stability of the interval polynomial.
1.0, \(q_1 = 1.0, q_3 = -0.43252, q_4 = -0.099186; c_{1,4} = 1.91360, q_1 = 1.0, q_2 = 0.5, q_3 = -0.37787, q_4 = -0.097877; c_{1,5} = 0.43664, q_1 = 1.0, q_2 = 0.5, q_3 = -0.40242, q_4 = -0.098962,\) in case:2, \{c_{1,3} = 2.14640, q_1 = 1.0, q_2 = 1.0, q_3 = -0.38546, q_4 = -0.098111; c_{1,3} = 2.34625, q_1 = 1.0, q_2 = 0.5, q_3 = -0.43297, q_4 = -0.099194; c_{1,4} = 2.46575, q_1 = 1.0, q_2 = 0.5, q_3 = -0.37002, q_4 = -0.097614; c_{1,5} = 0.60808, q_1 = 1.0, q_2 = 0.5, q_3 = -0.40208, q_4 = -0.098558\}. We know that each minimum of the element of Routh column is the positive real in case:1-2 and satisfies the robust stability criterion.1, the condition \(c_{1,i} = \min \{c_{1,i}(q)\} > 0 (i=1,2,3,4,5)\) is satisfied, so the interval matrix \(A\) in case:1-2 is a Hurwitz stable matrix and has robust stability in the prescribed set \(\Omega\). Based on robust stability criterion.3, the coefficients of interval polynomial in case:1-2 satisfy the necessary condition, the robust stability of the interval matrix \(A\) (also can be determined by the sequence \(Q_3\) or \(Q_4\) of the interval polynomial \(P(s,q)\)) as well. We can obtain the sequence \(Q_3\) or \(Q_4\) separately. In case:1, \(Q_2 = \{\min \{H_1(q_1), \min \{H_1(q_2)\}\} = \{2.14640, 9.37468\}, Q_3 = \{\min \{H_2(q_1), \min \{H_2(q_2)\}\} = \{5.09252, 8.03997\}, the sequence \(Q_3\) or \(Q_4\) in case:1-2 both satisfy the conditions \(Q_2 > 0\) or \(Q_3 > 0\) of the robust stability criterion.3, thus the interval matrix in case:1-2 is a Hurwitz stable matrix and has robust stability in the prescribed set \(\Omega\).

Example.3
Let us consider the interval matrix \(A\) of continuous-time linear system in [1], and the interval matrix has four uncertain parameters.

\[
A(q) = \begin{pmatrix}
q_1 & 0 & 0 & 0 \\
0 & q_2 & q_3 & 0 \\
0 & 0 & q_4 & q_5
\end{pmatrix}, q_1 \in [-2.478, -1.4471], q_2 \in [-0.0518, -0.0194], q_3 \in [2.00, 3.4347], q_4 \in [-0.0026, -0.0012], q_5 \in [-2.478, -1.4471], q_6 \in [-0.0518, -0.0194], q_7 \in [2.00, 3.4347], q_8 \in [-0.0026, -0.0012].
\]

Checking the robust stability of the interval matrix \(A\) can be equally transformed into the robust stability of an interval polynomial. So, the stability of the interval matrix \(A\) is equal to the stability of the interval polynomial \(P(s,q) = \det(sI - A(q))\), the corresponding interval polynomial of the interval matrix \(A\) can be acquired

\[
P(s,q) = \det(sI - A(q)) = s^3 - (q_1 + q_2 + q_3)s^2 + (0.7115q_1 + q_2q_4 + q_2q_4 + q_3q_5)s - q_1(q_2q_4 + 0.7115q_5) = s^3 + \sum a_is^i + H_1(q) = a_H - a_I;
\]

\[
H_1(q) = a_H = a, a_I = a.
\]

We are easily to check that all the coefficients \(a_i\) of the interval polynomial \(P(s,q)\) are the positive real, hence the coefficients satisfy the necessary condition of stable interval polynomial. Checking the global minimum of each Hurwitz determinant, or checking the global minimum of each element of Routh column, or checking the defined sequence \(Q_3\) or \(Q_4\) are the core tasks of determining the stability of the interval matrix \(A(q)\). So, the minimum of the Hurwitz determinants can be obtained in the set \(\Omega\): \(H_1(q) = 1.44770, q_3 = -1.4471, q_3 = -0.0194, q_3 = 3.4347, q_4 = -0.0012; H_2(q) = 0.073066, q_4 = -0.0012; H_3(q) = 0.150463, q_4 = -1.4471, q_3 = -0.0194, q_3 = 2.0, q_4 = -0.0012; H_4(q) = 0.150463, q_4 = -1.4471, q_3 = -0.0194, q_3 = 2.0, q_4 = -0.0012; H_5(q) = 0.150463, q_4 = -1.4471, q_3 = -0.0194, q_3 = 2.0, q_4 = -0.0012; the minimum of each Hurwitz determinant of the interval polynomial \(P(s,q)\) is the positive real, the stability condition \(H_i = \min \{H_i(q)\} > 0 (i=1,2,3)\) is satisfied, thus the interval polynomial \(P(s,q)\) is the Hurwitz stable polynomial, then we infer the corresponding interval matrix \(A(q)\) is the Hurwitz matrix and has the robust stability in the set \(\Omega\). When we apply the robust stability criterion.1, the minimum of each element of the Routh column will be acquired too, \(c_{1,2} = 6.00, q_1 = 1.00, q_2 = 1.00; H_2(q) = 6.00, q_3 = 1.00, q_3 = 1.00; H_3(q) = 36.00, q_4 = 1.00, q_4 = 1.00; H_4(q) = 36.00, q_4 = 1.00, q_4 = 1.00; H_5(q) = 36.00, q_4 = 1.00, q_4 = 1.00\); the minimum of each Hurwitz determinant of the interval polynomial \(P(s,q)\) is the positive real, the stability condition \(H_i = \min \{H_i(q)\} > 0 (i=1,2,3)\) is satisfied, thus the interval polynomial \(P(s,q)\) is the Hurwitz stable polynomial, and then we know that the corresponding interval matrix \(A(q)\) is the Hurwitz matrix and has the robust stability in the set \(\Omega\). When using the robust stability criterion.1, the minimum of each element of Routh column will be acquired too, \(c_{1,2} = 6.00, q_1 = \).
1.00 q \textsuperscript{1} = 1.00; c \textsuperscript{2} = 10.00, q \textsuperscript{1} = 1.00, q \textsuperscript{2} = 1.00; we know that each minimum of the element of Routh column is the positive real, the relationship $c_{ii}^{1} = \min\{c_{i1}(q)\} > 0$ $(i = 1, 2, 3, 4)$ is satisfied, so the interval matrix $A(q)$ is Hurwitz matrix. Furthermore, the robust stability of interval matrix can be verified only by checking the sequence $Q_3$ or $Q_4$ when using robust stability criterion.3. The coefficients of interval polynomial $P(s,q)$ satisfy the necessary condition, the stability of the interval matrix $A(q)$ also can be determined by the sequence $Q_3$ or $Q_4$ of the interval polynomial $P(s,q)$. We can obtain the sequence $Q_3$ or $Q_4$ respectively: $Q_3 = \{\min\{H_1(q), \min\{H_2(q)\}\} \} = \{6.00, 360.00\}$, and $Q_4 = \{\min\{H_i(q)\}\} = \{6.00\}$ satisfy the condition of sequence $Q_3 > 0$ or $Q_4 > 0$, thus the interval matrix is a Hurwitz stable matrix and has robust stability in the given set $\Omega$.

4. Assessing robust stability of discrete-time linear systems

**Inference.2:**

The robust stability criterions of interval polynomials of the continuous-time linear system have been presented in the section3, in this section, the stability of interval polynomials of the discrete-time linear system is introduced as well. Let us consider the interval polynomials of the discrete-time linear system, and we consider the following two types of interval polynomials

\begin{align*}
\text{case.1: } P(z,a) &= z^i + a_{i1}z^{i-1} + \ldots + a_{i0}, z = z^i + \sum_{x=1}^{i}a_xz^{i-x}, a \in [a_{i0}, a_{i1}] \\
\text{case.2: } P(z,q) &= z^i + \sum_{x=1}^{i}f_x(q,q_i,q_{i1}, \ldots, q_{ik})z^{i-x} = z^i + \sum_{x=1}^{i}a_xz^{i-x}, M_i \leq a_x \leq M_i', q \in [q_0(q_i), q_{1i}, \ldots, q_{ik}],
\end{align*}

Where $R$ is the real set, $a_i$ is the uncertain coefficient of the interval polynomial, and $q_i$ is the uncertain parameter of the interval polynomial respectively. Then the robust stability of the interval polynomials of discrete-time linear system in the case.1 and case.2 can be ultimately equally transformed into the following robust stability of interval polynomials $P(z,a)$ and $P(z,q)$, or $P(z,a)$ and $P(z,w)$ through bilinear transformation $z = (w+1)/(w-1)$.

\begin{align*}
\text{case.1: } P(z,a) \in S(z) \Rightarrow P(w,a) \in H(w) \\
\text{case.2: } P(z,q) \in S(z) \Rightarrow P(w,q) \in H(w)
\end{align*}

Where $S(z)$ is the set of all Schur-Cohn stable polynomials of discrete-time linear system, $P(w,a)$ and $P(w,q)$, or $P(w,a)$ and $P(w,q)$ are the new interval polynomials, the detail deviations of the equally transformed process of the interval polynomial $P(z,a)$ and $P(z,q)$ in the case.1 and case.2 are presented in the Appendix.2, thus the robust stability of interval polynomials of the discrete-time linear system can be determined by the robust stability checking methods of the continuous-time linear system, which is proved by the studied examples in this section. In the case.2, the bounded real polynomial coefficient $a_i$ of the interval polynomial depends on multi-affine continuous mapping of uncertain parameters $q_i$.

**Remark.5:** The sufficient and necessary conditions of the discrete-time linear invariant system (DTLIS) if and only if are that: all the poles of characteristic polynomial or equation of discrete-time linear invariant system should be in unit circle that should satisfy the relationships $|z| < 1$ $(i = 1, 2, \ldots, n)$ [27], then we call the control system Schur-Cohn stability. As we known, the Schur-Cohn criterion and Jury criterion in [27] are usually used to check the stability of the DTLIS. Besides the Schur-Cohn criterion and the Jury criterion, the Routh-Hurwitz criterion also can be used to check the stability of the DTLIS through bilinear transformation. In order to use the Routh-Hurwitz criterion and avoid checking all the poles within in the unit circle, the bilinear transformation is applied to transform the stability of the DTLIS. The unit circle region is mapped to the left half of complex plane by using bilinear transformation $z = (w+1)/(w-1)$. Thus, checking the stability of the DTLIS can apply the stability criterions and methods of the continuous-time linear invariant system (CTLIS). Motivated by this idea, thus the robust stability of the interval polynomials of discrete-time linear system also can be transformed into continuous-time linear system. If the interval polynomial $P(z,a)$ and $P(z,q)$ are the Schur-Cohn stable polynomial, then all the poles of interval polynomial $P(z,a)$ and $P(z,q)$ should satisfy relationships $|z| < 1$ $(i = 1, 2, \ldots, n)$. However, Schur-Cohn criterion and Jury criterion are not very suitable to directly check the stability of interval polynomial of the discrete-time linear system. Directly checking the stability of the interval polynomials or matrices of discrete-time linear system may be a difficult work. In the section.4, we check the robust stability of the interval polynomials of the discrete-time linear system is equally transformed into the robust stability of continuous-time interval polynomial by applying bilinear transformation. Thus, the robust stability theorem.1-3 can be extended into discrete-time linear system by applying bilinear transformation. Once the robust stability of the interval polynomial $P(w,a)$ and $P(w,q)$, or $P(w,a)$ and $P(w,q)$ are determined, and robust stability of the corresponding interval polynomial $P(z,a)$ and $P(z,q)$ are fixed as well. In the section.4, the studied examples show that the robust stability theorem.1-3 can be used to check to robust stability of interval polynomial of the discrete-time linear system.

4.1 Numerical examples of discrete time systems

**Example.5**

Let us consider following interval matrix $A(q)$ of discrete-time linear system in [33], and the interval matrix contains two uncertain parameters.

\begin{align*}
A(q) &= \begin{pmatrix}
-0.7 & 0 & 0 & 0.4 \\
-0.3 & -0.7 & 0.4 & 0.3 \\
0.3 & 0.4 & -0.7 & 0.4 \\
-0.1 & -0.4 & -0.3 & -0.1
\end{pmatrix} + q_i \begin{pmatrix}
0.7 & 0.7 & 0.5 & 0.4 \\
0.7 & 0.7 & 0.5 & 0.4 \\
-1.5 & 1.0 & 0.7 & 0.4 \\
-1.5 & 1.0 & 0.7 & 0.4
\end{pmatrix} + q_j \begin{pmatrix}
-1.2 & -1.0 & -0.6 \\
-1.2 & -1.0 & -0.6 \\
-1.2 & -1.0 & -0.6 \\
-1.2 & -1.0 & -0.6
\end{pmatrix}
\end{align*}

Case.1: $q_i \in [0.30, 0.30], q_j \in [-0.30, 0.30], \Omega = [0.30, 0.30] \times [-0.30, 0.30]$.

Case.2: $q_i \in [0.35, 0.35], q_j \in [-0.38, 0.38], \Omega = [0.35, 0.35] \times [-0.38, 0.38]$.

Case.3: $q_i \in [-0.40, 0.40], q_j \in [-0.50, 0.50], \Omega = [-0.40, 0.40] \times [-0.50, 0.50]$.

The interval polynomial $P(z,q)$ of interval matrix $A(q)$ is

\begin{align*}
P(z,q) &= \text{det}(A(z,A(q))) = (z^3 - 0.7 - 0.7q_1 + 0.7q_2)z + (0.7q_1 + 0.07 - 1.28q_3 + 0.47q_2 + 0.37q_1 + 1.3q_2)z + (0.502q_1 - 0.084q_2 - 0.296q_3 - 0.15q_4 + 0.695q_2 + 0.008q_1^2 - 1.62q_3^2 - 0.726q_2^2 - 1.678q_1q_2 - 0.042) = z^3 + \sum_{x=1}^{3}a_xz^{x-1}
\end{align*}
According to the inference, the robust stability of the interval polynomial \( P(z, q) \) can be transformed into following stability of the interval polynomial \( P_i(w, q) \):

\[
P_i(w, q) = \frac{\sum_{i=1}^{n} a_i w^i}{\sum_{i=1}^{n} b_i w^i} = \frac{\sum_{i=1}^{n} c_i(w)}{\sum_{i=1}^{n} d_i(w)}
\]

Based on the robust stability criterion, the global minimum of the Hurwitz determinant of interval polynomial \( P_i(w, q) \) can be obtained in case.1-3 respectively. In case.1: \( |H_1(q)| = 0.74050 \), \( q^* = -0.30 \), \( q^* = 0.30 \); \( H_1(q) = 0.08641, q^* = -0.30, q^* = 0.30 \), \( q_1(q) = 0.0084616, q^* = -0.30, q^* = 0.30 \); in case.2: \( |H_1(q)| = 0.4114, q^* = -0.35, q^* = 0.35 \); \( H_1(q) = 0.055856, q^* = -0.35, q^* = 0.35 \); \( H_1(q) = 0.016019, q^* = -0.35, q^* = 0.35 \); in case.3: \( |H_1(q)| = 0.021866, q^* = -0.40, q^* = 0.50 \); \( H_1(q) = -0.26985, q^* = -0.40, q^* = 0.50 \); \( H_1(q) = -0.067559, q^* = -0.40, q^* = 0.50 \). We know that each minimum of the Hurwitz determinant \( H(q) \) of interval polynomial \( P_i(w, q) \) is the Hurwitz stable polynomial in the set \( \Omega_1 \) and \( \Omega_2 \), according to the inference, the interval polynomial \( P_i(z, q) \) of interval matrix \( A(q) \) is the Schur-Cohn stable polynomial in the set \( \Omega_1 \) and \( \Omega_2 \), so the corresponding interval matrix \( A(q) \) of discrete-time linear system is a Schur-Cohn matrix and has the robust stability in the set \( \Omega_1 \) and \( \Omega_2 \). But, the interval polynomial \( P_i(w, q) \) in the set \( \Omega_1 \) is not a Hurwitz stable polynomial because each minimum of the Hurwitz determinant does not satisfy stable condition \( H_1 = \min \{ |H(q)| \} > 0 \) (i = 1, 2, 3). Thus, the interval matrix \( A(q) \) and its interval polynomial \( P_i(z, q) \) does not belong to Schur-Cohn stable matrix and polynomial in the set \( \Omega_3 \) respectively. Likewise, when we use the robust stability criterion, the minimum of each element of Routh column will be acquired too. In case.1: \( |c_{1,2}(q)| = 0.74050, q^* = -0.30 \), \( q^* = 0.30 \); \( c_{1,2}(q) = 0.41680, q^* = -0.30 \), \( q^* = 0.30 \); \( c_{1,2}(q) = 0.11036, q^* = -0.30 \), \( q^* = 0.30 \) in case.2: \( |c_{1,2}(q)| = 0.41141, q^* = -0.35 \), \( q^* = 0.35 \); \( c_{1,2}(q) = 0.13423, q^* = -0.35 \), \( q^* = 0.35 \); \( c_{1,2}(q) = 0.09654, q^* = -0.35 \), \( q^* = 0.35 \) in case.3: \( |c_{1,2}(q)| = 0.021866, q^* = -0.40 \), \( q^* = 0.50 \); \( c_{1,2}(q) = -0.1944944, q^* = -0.38172, q^* = 0.50 \); \( c_{1,2}(q) = 0.073681, q^* = -0.40 \), \( q^* = 0.50 \). We know that each minimum of the element of Routh column in the set \( \Omega_1 \) and \( \Omega_2 \) is the positive real, the stable condition \( c_{i,1} = \min \{ c_{i,1}(q) \} > 0 \) (i = 1, 2, 3, 4) is satisfied, so the interval polynomial \( P_i(w, q) \) is a Hurwitz stable polynomial in set \( \Omega_1 \) and \( \Omega_2 \). According to the inference, the interval matrix \( A(q) \) and interval polynomial \( P_i(w, q) \) in set \( \Omega_1 \) and \( \Omega_2 \) are the Schur-Cohn stable matrix and polynomial respectively, thus they have the robust stability in set \( \Omega_1 \) and \( \Omega_2 \). But, the interval polynomial \( P_i(w, q) \) is not a Hurwitz stable polynomial in set \( \Omega_2 \) because stable condition \( c_{1,1} = \min \{ c_{1,1}(q) \} > 0 \) (i = 1, 2, 3, 4) is not satisfied, of course we can infer that the interval matrix \( A(q) \) and its interval polynomial \( P_i(z, q) \) are not the Schur-Cohn stable matrix and polynomial in set \( \Omega_2 \) respectively.

5. Conclusion

The paper focuses on study the interval polynomials and interval matrices’ robust stability of the continuous- and discrete-time linear system which are affected by the real uncertain parameters. The robust stability of the interval polynomials and matrices can be determined through three robust stability criterions that check the minimum of each Hurwitz determinant or the minimum of the Routh column each element in the given set. The third robust stability criterion can reduce computational complexity and the number of the optimization objectives when we determine the robust stability of the interval polynomials. The robust stability of the interval matrices can be equally transformed into the stability of an interval polynomial. The three robust stability criterions can be extended into the discrete-time linear system through the bilinear transformation. Different examples of the continuous-time system and discrete-time system’s interval polynomials and matrices are discussed to demonstrate the effectiveness and accurateness of robust stability checking methods.

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References


**Appendix 1.**

\[ H_2(a) = \begin{vmatrix} a_1 & a_2 & \cdots & a_n \\ a_1 & a_2 & \cdots & a_n \\ \vdots & \vdots & \ddots & \vdots \\ a_1 & a_2 & \cdots & a_n \end{vmatrix} \]

where \( a_i = 0 \) for \( i > a \)

\[ H_2(a) = \begin{vmatrix} H(a) & H(a) & \cdots & H(a) \\ H(a) & H(a) & \cdots & H(a) \\ \vdots & \vdots & \ddots & \vdots \\ H(a) & H(a) & \cdots & H(a) \end{vmatrix} \]

\[ Q_1 = \{ \forall a \in \Omega, \exists \psi \in \Omega \} \]

\[ Q_2 = \{ \forall \psi \in \Omega \} \]

**Appendix 2.**

\[ P(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_1z + a_0 \]

\[ P(z) = \sum_{i=0}^{n} a_i z^i \]

\[ Q = \{ \forall a \in \Omega \} \]

\[ Q_1 = \{ \exists \psi \in \Omega \} \]

\[ Q_2 = \{ \forall \psi \in \Omega \} \]