Selective Method of Wavelet Basis Function and Application in Signal Processing of EGG

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Abstract—In this paper, signal energy, information entropy and joint entropy were analyzed to introduce general selective method of wavelet basis function, and these methods were applied in slow wave diagnosis of electrogastrogram(EGG). In order to verify the rationality of evaluation criteria, EGG signals were processed and filtered with different wavelet basis function in the experiments. Experimental results show that db9 wavelet could achieve the most energy of filtered signal, by the EE/ET composite index, db5 wavelet function could achieve the best result. Meyer wavelet could realize the most similar index. Power spectrum of EGG was used to prove the effectiveness of selective method of wavelet basis function, and effective selection criteria were also provided to select wavelet basis function in wavelet transform.

Keywords- electrogastrogram; wavelet transform; wavelet basis function; entropy; power spectrum.

I. INTRODUCTION

Wavelet basis function is a function trending to zero with attenuation and fluctuation. In 1910, A. Haar proposed the concept of orthogonal basis of Haar wavelet. In 1986, Meyer constructed a smooth function with attenuation, and the binary orthonormal basis was also constructed. Daubechies constructed a continuous orthogonal wavelet basis from the perspective of scaling function, and applications of wavelet theory were promoted to a new climax [1]. In 1988, S. Malla proposed a multi-resolution analysis method (MRA, Multi-Resolution Analysis), and the structuring method of orthonormal wavelet and fast algorithm were also given. Wavelet functions have different characteristics, and using of different wavelet functions would achieve different results in signal processing. Therefore, selective optimization of wavelet function becomes a research-worthy problem, and some optimization methods and results have been done in recent years [2]-[7].

As is well known, gastric motility of human is associated with action potential. In the normal human stomach, digestive actions generate circadian rhythm of stomach [8]-[9]. Gastric electrical activities recorded with skin surface electrodes on the abdominal gastric wall at corresponding location inside the stomach is called electrogastrogram. Regular electrical fluctuations in the stomach are consisted of gastric slow waves, or electrical control behaviors and spike wave potential, or electrical response behaviors. Gastric slow wave (slow wave in gastric electrogastrogram) is an electrical activity in the stomach on a cyclical change, or basic electric rhythm (basic electrical rhythm, BER), and the frequency in the human body is 0.05Hz [10]. Spike wave of EGG (spike wave in gastric electrogastrogram) is also known as action potential. Spike potentials of gastric smooth muscle are often located on slow wave [11].

Wavelet transform(WT) is used to solve the problem of non-stable signal such as EGG. Wavelet transform has applied in biological signal processing for many years. A series of shifted and compressed wavelet basis functions are used to mapping sampling signal. WT can be divided into CWT (continuous wavelet transform) and DWT (discrete wavelet transform). This paper used a variety of wavelet functions to process experimental data, and information entropy, joint entropy and signal energy of wavelet function were used to evaluate the optimal choice of wavelet basis function.

II. WAVELET TRANSFORM AND WAVELET BASIS FUNCTIONS

The meaning of wavelet transform is that a series of translated and compressed wavelet function are used to describe sampling signal. Signal analysis is usually used in order to obtain the relationship between time domain and frequency domain. Information characteristic of time is obtained through shifting the signal, and the frequency characteristic of the signal is available through scaled transform of WT [12].

The continuous wavelet transform of a signal \(x(t)\) is defined as,

\[
WT_{x}(a,\tau) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t)\psi^{*} \left(\frac{t-\tau}{a}\right)dt \quad , \quad a > 0
\]  

(1)

where \(x(t)\) is the sample signal and \(\psi^{*}(\cdot)\) is the wavelet basis function. CWT can be expressed in frequency domain as following.

\[
WT_{x}(a,\omega) = \frac{\sqrt{a}}{2\pi} \int_{-\infty}^{\infty} X(\omega)\psi^{*}(a\omega)e^{i\omega\tau} d\omega
\]  

(2)
Wavelet transform of discrete signal with respect to the wavelet $\psi(t)$ could be discretized as follows,

$$CWT(T_i, a) = T_i \sqrt{a} \sum_{n}^{} x(n T_i) \psi^* \left( \frac{(n-i)T_i}{a} \right)$$  \hspace{1cm} (3)

where $T_i$ is the sampling interval, and $i$ is the integer sample number.

Wavelet transform which using a scaling factor and translation parameters is called the two-scale wavelet transform, which is a form of discrete wavelet transform (DWT). In this paper, sampling signal is decomposed into 9 layers with discrete wavelet analysis, and then Fourier transform is also used to get accurate signal in slow wave bandwidth (0.040-0.062Hz). The information entropy, signal energy and joint entropy of slow wave were calculated to evaluate wavelet basis function, and those were used to optimize choice of wavelet basis function.

Wavelet basis functions have many families which include Haar, Daubechies, Symlet, Coiflet, BiorSpline, ReverseBior, Meyer, Gaussian, Mexican hat, Morlet, Shannon, Frequency B-Spline etc, as shown in Fig1-3.

The characteristics of wavelet basis function are as follows. The orthogonality is that the inner product of wavelet basis function with itself is 1. The inner product of wavelet function with a scaled and shifted wavelet is zero.

Wavelet basis function with compact support is only non-zero in a limited range which is the support width of wavelet basis function. The computational complexity would be lower with the smaller support width.

The regularity is generally used to describe the smoothness of wavelet basis function. The better the regularity is, the more smooth wavelet basis function would be, and the faster the convergence would be. Lipschitz exponent $\alpha$ is usually used to measure the regularity.

The convergence rate of wavelet function approaching smooth function is determined by the disappearance distance of wavelet basis function. A large distance of disappearance is generally needed in wavelet decomposition.

The selective principles of wavelet basis function are as follows. If wavelet basis function has a certain similarity with sampling signal, the energy of filtered signal would be more concentrated and the complexity of calculation would be reduced after transformation. The discriminant function is to find some technical indexes and a class of discriminant functions for certain types of key issues, the wavelet functions are substituted into indexes.

### III. INDEX OF EVALUATION

Maximum energy of the wavelet based function selective method is proposed in this paper. Energy is an important feature of signal, and it has close relationship with amplitude of signal. The energy of sampling signal is as follows,

$$E(x) = \sum_{i=1}^{N} |x(i)|^2$$  \hspace{1cm} (4)

If there is a signal with certain frequency bandwidth in a signal, the bandwidth will have higher amplitude after wavelet transform, and the signal energy corresponding to the frequency components can be extracted from the signal. More energy are extracted, wavelet basis function will be more effective.

Information entropy is the basis of modern information theory. Entropy represents of the amount of information, and entropy is a measure of uncertainty. The greater it is, less information of the problem is.

Energy distribution of signal is quantitatively described by Shannon entropy. $X$ is a discrete random variable defined in probability space $(\Omega, F, P)$, a probability test with n-uncertain
outcomes, we defined the results $(x_1, x_2, \ldots, x_n)$ which have discrete probability distribution $p_i (i=1, 2, \ldots, n)$, entropy of probability measure is defined as follows,

$$H_s(W) = -\sum_{i=1}^{N} p_i \times \log_2 p_i$$

(5)

$p_i$ is the probability density of energy distribution. The smaller the entropy is, the greater the amounts of information are. Wavelet coefficients of a better wavelet basis function generate some large values after wavelet decomposition, and amplitudes ignored in other coefficients so as to obtain the minimum Shannon entropy.

$$R(s) = \frac{E_s(W)}{H_s(W)}$$

(6)

Maximizing the ratio of the energy-Shannon entropy is reasonable as a complex index for selecting wavelet basis function. Selective method of wavelet function based on maximum similarity is explained as follows. Sampled signal and wavelet function variable are random variables, the density function of joint probability of two variables is $p(x,y)$, joint entropy which is one of similarity index is described as formula (7). The similarity is greater with smaller joint entropy.

$$H(X,Y) = -\sum_{x\in X} \sum_{y\in Y} p(x,y) \log p(x,y)$$

(7)

IV. EXPERIMENTS OF EGG AND DATA ANALYSIS

Experimental system included skin surface electrodes, amplifier with two channels (500X, filter with 20Hz lowpass), data collective card with eight channels (NI BNC-2110) and data processing PC, as shown in Figure 4. Reference electrode $R$ is located in xiphoid. A3 is located in mid-point of the xiphoid and the navel. A1 and A2 are about 45 degree angle to horizontal line, the distance is approximately 4-5cm. Two-channels scheme is adopted in these experiments, so we only take positions R, A1 and A3, as shown in Fig.5. Talking and moving were not allowed during experiments, and testees were suggested to sit comfortably to maintain stable and prevent from movements. The recordings of EGG data were still going on without getting off electrodes during test meal. Each sampling time was about 80-100 minutes.

![Fig.4 Experimental system of EGG](image)

![Fig.5 Positions of skin sampling electrodes](image)

Decompositions with nine layers were adopted in this paper, as shown in Figure 6. Sampling frequency of the EGG signal is 100Hz. The frequency range is halved of the bandwidth of original signal after decomposition. The bandwidth of slow wave is 0.040-0.062Hz, learned from the wavelet signal decomposition figure, the bandwidth of A9 is 0.040-0.062Hz. The table below compares energy, entropy and EE/ET of EGG signal.

<table>
<thead>
<tr>
<th>Wavelet function</th>
<th>Energy</th>
<th>Entropy</th>
<th>EE/ET</th>
</tr>
</thead>
<tbody>
<tr>
<td>Db2</td>
<td>13.067524</td>
<td>1.84749</td>
<td>7.0731</td>
</tr>
<tr>
<td>Db4</td>
<td>13.180134</td>
<td>1.84365</td>
<td>7.1489</td>
</tr>
<tr>
<td>Db5</td>
<td>13.327843</td>
<td>1.84726</td>
<td>7.2149</td>
</tr>
<tr>
<td>Db6</td>
<td>13.305536</td>
<td>1.85061</td>
<td>7.1898</td>
</tr>
<tr>
<td>Db8</td>
<td>13.274707</td>
<td>1.84807</td>
<td>7.1830</td>
</tr>
<tr>
<td>Db9</td>
<td>13.333622</td>
<td>1.84918</td>
<td>7.2105</td>
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<tr>
<td>Db10</td>
<td>13.301901</td>
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<tr>
<td>Coif1</td>
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<td>1.85086</td>
<td>6.9965</td>
</tr>
<tr>
<td>Coif3</td>
<td>13.256546</td>
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<tr>
<td>Sym2</td>
<td>13.067524</td>
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<td>7.0731</td>
</tr>
<tr>
<td>Sym4</td>
<td>13.186869</td>
<td>1.85061</td>
<td>7.1256</td>
</tr>
<tr>
<td>Sym6</td>
<td>13.242703</td>
<td>1.85067</td>
<td>7.1556</td>
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<tr>
<td>Meyer</td>
<td>13.258869</td>
<td>1.84959</td>
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<td>Bior1.3</td>
<td>13.025240</td>
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<td>Bior4.4</td>
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<td>Rhio5.5</td>
<td>13.236723</td>
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</tr>
</tbody>
</table>
0.0-0.098 Hz. Frequency bandwidth A9 was filtered through the wavelet transform, and spike wave bandwidth was applied with inverse-Fourier transform, a precise filter of slow wave frequency was achieved finally. Frequency spectrum and power spectrum of fasting, postprandial slow wave were quantitatively analyzed to obtain slow-wave signal of gastric motility.

The sampling signals were filtered with different wavelet functions respectively. The signal energy, Shannon entropy and joint entropy of the obtained slow wave signals were calculated as Table I and II.

By comparison of data Table I and II, the db9 wavelet could be used to obtain the maximum energy of filtered signal; db4 wavelet obtained the minimum Shannon entropy of filtered signal; Db5 wavelet could obtain the maximum ratio of energy and entropy; and Meyer could obtain the maximum joint entropy.

Selecting method based on ratio of energy and entropy was also validated to be valid. At last, detection method of slow wave was verified to be effective.

V. CONCLUSIONS

Wavelet analysis has been widely applied in biological signal processing. Signal processing and filter result would be different with various wavelet functions. As a result, how to select a wavelet function became a difficult problem. Through analysis of information entropy, signal energy and joint entropy of wavelet function, some general selective criteria were introduced and were applied in the signal processing of EGG in our paper. Experimental results and analysis show that selective methods based on information entropy, signal energy and joint entropy of wavelet function were proved to be effective through power spectrum of EGG.

ACKNOWLEDGMENT

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REFERENCES


