Pose Determination from One Point and Two Coplanar Line Features*

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Abstract. Monocular pose determination from point and line mixed features is a typical problem in computer vision. Given one point feature and two coplanar line features on an object and their correspondences on the image plane of a calibrated camera, the pose parameters between the camera and the object can be calculated. The problem is studied in two cases that two lines are parallel or not. The solution number properties of the problem are proved according to the geometric relationship of the three features, and generally it has two solutions at most. The closed form solution of the problem is also presented. The results provide a new method for pose determination using monocular vision.

Keywords: pose determination, monocular vision, point feature, line feature.

1 Introduction

Computer vision based pose measurement has been widely used in industry automatic assembling. In generally cases, we can adopt monocular vision method or binocular vision method. Compared with binocular method, the monocular method has advantage of simpler system structure, but it needs to know some features, such as point features or line features, on the target as cooperative features. Applying point features, the most famous method is Perspective-n-Point (PnP) method [1], which uses n control points and their correspondences on the image to calculate the position and orientation between the target and the camera. If $n \le 2$, the pose parameters can not be obtained from known conditions. If $n > 5$, the problem can be solved by linear method. If $3 \le n \le 5$, PnP problem generally is a nonlinear problem and has multiple solutions. In general cases, the P3P problem has four solutions [1]. Wang et al. found when three control points construct an isosceles triangle and the optical center of the camera locates inside seven special regions, the problem has unique solution [2]. While applying straight line features, the similar problem is called PnL (Perspective-n-Line) problem. Dhome et al.

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found general P3L problem has eight solutions, and the real solution needs other conditions to distinguish [3]. Qin et al. studied a special case of P3L that three lines intersect at two points and found the problem has four solutions [4]. Liu et al. studied other special cases of P3L and found that if three lines are geometric Z-shaped, the problem's solution number depends on the location of the optical center of the camera, and if the optical center locates inside a special region, the problem has unique solution [5]. Ying et al. studied a case that three lines are all parallel and got two solutions [6]. Shi et al. studied a case that three lines intersect at one point (it is also called corner feature) and got unique orientation solution [7].

Above research works all depend on simply point features or line features. In real world, point and line mixed features exist widely, but related research works, such as solving conditions, closed form solutions, solution number properties are seldom studied. In this paper, we present a pose determination method using point and line mixed features. Suppose the camera is calibrated, and given one point and two coplanar line features on the target and their correspondences on the image, the position and orientation between the target and the camera can be calculated.

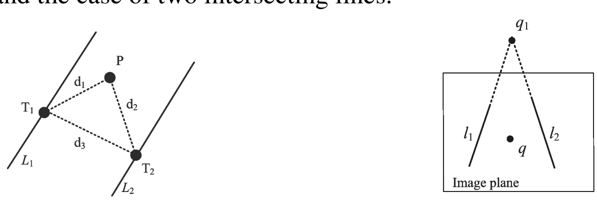
2 Camera Model and Problem Statement

In this paper we adopt pin-hole model for a camera. The camera's four intrinsic parameters include principal point (u_0, v_0) and the focal ratio f_u and f_v where $f_u = f / d_u$ and $f_v = f / d_v$, where f is the focal length, d_u and d_v are the pixel distance. The camera's extrinsic parameters include: rotation matrix R and translation vector $T = (T_X, T_Y, T_Z)$ where R can be expressed by three rotation angles around three axes. If a point has the world coordinates (x_W, y_W, z_W) and the camera coordinates (x_C, y_C, z_C) at the same time, two coordinates satisfy

$$\begin{pmatrix} x_C & y_C & z_C \end{pmatrix} = \begin{pmatrix} x_W & y_W & z_W \end{pmatrix} R + T , \quad (1)$$

where R is the rotation matrix and T is the translation vector. Suppose the camera is calibrated, all intrinsic parameters of the camera are known, and also given the correspondences of the three features on the target and their image features. The problem of pose determination is to find out the transformation between the world and camera coordinates, that is the value of R and T in (1).

In this paper, we discuss the problem in the following two cases, the case of two parallel lines, and the case of two intersecting lines.



(a) three features (b) image of the three features

Fig. 1. Problem description of two parallel lines

3 The Case of Two Parallel Lines

3.1 Problem Description

In the case that two lines are parallel, the problem is illustrated as Fig. 1. The relationship of three features are shown as Fig. 1(a), two straight lines L_1, L_2 and point P are three given features. L_1 and L_2 are parallel, P is not on L_1 or L_2. Suppose $PT_1 \perp L_1$ at T_1 and $PT_2 \perp L_2$ at T_2, $|PT_1| = d_1$, $|PT_2| = d_2$, $|T_1T_2| = d_3$. The image of the three features is shown as Fig. 1(b), the image of L_1 and L_2 are l_1 and l_2, the image of point P is q. We do not discuss the degenerated case, it means l_1 and l_2 are not point, and q is not on l_1 or l_2. The problem can be described to find out the coordination of P, the line functions of L_1 and L_2 in the camera coordinate system.

3.2 Solution Properties and Closed Form Solution

If the camera's image plane is not parallel to L_1 and L_2, l_1 is not parallel to l_2. Suppose the intersection point of l_1 and l_2 is q_1, which is called the vanishing point of L_1 and L_2. Because the connection line of the camera's optical center and q_1 is parallel to L_1 and L_2, we can get the direction of L_1 and L_2 in the camera coordinate system. If the camera's image plane is parallel to L_1 and L_2, we have $l_1 // l_2 // L_1$, suppose q_1 is any point such that $O_C q_1 // l_1$, we can also get the direction of L_1 and L_2.

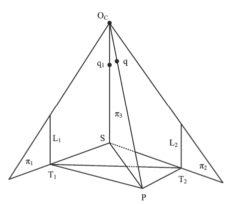


Fig. 2. Solution properties in the case of parallel lines

See as Fig. 2, suppose the camera's optical center is O_C, the planes defined by O_C and l_i are π_i $(i = 1,2)$. From $O_C q_1 // L_1 // L_2$, we have $O_C q_1$ must be the intersection line of π_1 and π_2. Because the visible part of L_1 must locate in front of the camera, we have L_1 must locate in a half-plane of π_1. We also use π_1 to denote this half-plane, so as π_2. Suppose the half-plane defined by q and $O_C q_1$ is π_3. From the pin-hole model of the camera, P must locate on the ray $\overrightarrow{O_C q}$. Suppose the intersection point of $O_C q_1$ and the plane PT_1T_2 is S; the standard normal vectors of π_i are $N_i = (n_{ix}, n_{iy}, n_{iz})^{\mathrm{T}}$ $(i = 1,2,3)$. The problem is transformed to find out the coordinates of P, T_1 and T_2 in the camera coordinate system.

Suppose the angle between π_1 and π_3 is α, the angle between π_2 and π_3 is β, the angle between π_1 and π_2 is γ $(0<\alpha,\beta,\gamma<\pi)$. The value of α, β, γ can be calculated from N_1, N_2 and N_3. Because L_1 is perpendicular to the plane PT_1T_2, and $O_Cq_1//L_1$, we have O_CS must be perpendicular to the plane PT_1T_2, then we have $\angle T_1SP=\alpha$, $\angle T_2SP=\beta$, $\angle T_1ST_2=\gamma$.

We firstly calculate the location of S from P, T_1 and T_2. From $|PT_1|=d_1$, $|PT_2|=d_2$ and $|T_1T_2|=d_3$, the problem of location of S is transformed into a degenerated P3P problem, which has following three constraints

$$\begin{cases}|SP|^2+|ST_1|^2-2\cos\alpha\cdot|SP||ST_1|=|PT_1|^2=d_1^2\\|SP|^2+|ST_2|^2-2\cos\beta\cdot|SP||ST_2|=|PT_2|^2=d_2^2\\|ST_1|^2+|ST_2|^2-2\cos\gamma\cdot|ST_1||RT_2|=|T_1T_2|^2=d_3^2\end{cases}. \tag{2}$$

Suppose the coordinates of point P, T_1 and T_2 on the plane PT_1T_2 are (x_3,y_3), (x_1,y_1) and (x_2,y_2) respectively, the coordinates of point S is (x,y), (2) can be transformed into the following equation system

$$\begin{cases}(x-x_{13})^2+(y-y_{13})^2=r_{13}^2 \text{ or } (x-x'_{13})^2+(y-y'_{13})^2=r_{13}^2\\(x-x_{23})^2+(y-y_{23})^2=r_{23}^2 \text{ or } (x-x'_{23})^2+(y-y'_{23})^2=r_{23}^2\\(x-x_{12})^2+(y-y_{12})^2=r_{12}^2 \text{ or } (x-x'_{12})^2+(y-y'_{12})^2=r_{12}^2\end{cases}, \tag{3}$$

where

$$\begin{cases}x_{13},x'_{13}=\dfrac{x_1+x_3}{2}\pm\dfrac{y_1-y_3}{2\tan\alpha};\ y_{13},y'_{13}=\dfrac{y_1+y_3}{2}\mp\dfrac{x_1-x_3}{2\tan\alpha};\ r_{13}=\dfrac{\sqrt{(x_1-x_3)^2+(y_1-y_3)^2}}{2\sin\alpha}\\x_{23},x'_{23}=\dfrac{x_2+x_3}{2}\pm\dfrac{y_2-y_3}{2\tan\beta};\ y_{23},y'_{23}=\dfrac{y_2+y_3}{2}\mp\dfrac{x_2-x_3}{2\tan\beta};\ r_{23}=\dfrac{\sqrt{(x_2-x_3)^2+(y_2-y_3)^2}}{2\sin\beta}\\x_{12},x'_{12}=\dfrac{x_1+x_2}{2}\pm\dfrac{y_1-y_2}{2\tan\gamma};\ y_{12},y'_{12}=\dfrac{y_1+y_2}{2}\mp\dfrac{x_1-x_2}{2\tan\gamma};\ r_{12}=\dfrac{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}{2\sin\gamma}\end{cases}.$$

This equation system is to find out the common intersection point of three circles, it is a quadratic equation. Regarding the number of degenerated P3P problem, we have following lemma [6].

Lemma 1: If P, T_1, T_2 and S four points are all on a circle, the degenerated P3P problem has infinite solutions; if S is inside or on the edge line of the triangle PT_1T_2, the problem has a unique solution; otherwise the problem has two solutions.

From (3), we can get the location of S on the plane PT_1T_2, then we can calculate the distance from S to point P, T_1 and T_2, they are $|PS|$, $|T_1S|$ and $|T_2S|$. We have

$$|O_C P| = \frac{|PS|}{\sin(\angle q_1 O_C q)}.$$

Then we can get the coordinates of point P in the camera coordinate system. It is

$$\overrightarrow{O_C P} = \frac{\overrightarrow{O_C q}}{|\overrightarrow{O_C q}|} \cdot |O_C P| = \frac{|PS|}{\sin(\angle q_1 O_C q) \cdot |\overrightarrow{O_C q}|} \cdot \overrightarrow{O_C q}.$$

From the value of $|PT_1|$ and $|PT_2|$, we can also get the coordinates of T_1 and T_2 in the camera coordinate system, then we can get the line equations of L_1 and L_2 in the camera coordinate system.

From Lemma 1, we have the following theorem regarding the solution number of the problem in the case of parallel lines.

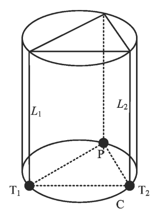


Fig. 3. Relationship between the solution number and the optical center's location

Theorem 1: Regarding the problem described in Fig. 1, the solution number of the problem depends on the location of the optical center and three features. See as Fig. 3, suppose C is the circumcircle of the triangle PT_1T_2, if the optical center locates on the cylinder whose base circle is C, the problem has infinite solutions; if the optical center locates inside the triangular prism whose base triangle is PT_1T_2, the problem has unique solution; if the optical center locates on the surface of the triangular prism, the problem is degenerated; otherwise the problem has two solutions.

It is very interesting that the cylinder in Fig. 3 is just the "Dangerous Cylinder" in P3P problem [8]. It shows in this case the problem has similar properties to P3P problem.

4 The Case of Two Intersecting Lines

4.1 Problem Description

In the case of two intersecting lines, the problem is illustrated as Fig. 4. Two lines L_1, L_2 and point P are three given features. Suppose the intersection point of L_1 and L_2 is P_0, point P is not on L_1 or L_2. Suppose L_3 is the line connects P_0 and P, and suppose L_1, L_2 and L_3 are all rays start from P_0. The angle between L_i and L_j is α_{ij} $(i, j = 1,2,3)$, and $|P_0P| = d$. The image line of L_1, L_2 and L_3 are l_1, l_2

and l_3 respectively, the image point of P is q, the image point of P_0 is q_0. We do not discuss the degenerated case, it means l_1, l_2 and l_3 are not points, q is not on l_1 or l_2. In this case, the pose determination problem can also be transformed to find out the coordinates of P_0, P and the line functions of L_1, L_2 and L_3 in the camera coordinate system from their images.

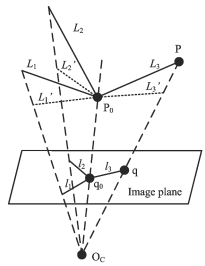


Fig. 4. Problem description of two intersecting lines

4.2 Solution Properties and Closed Form Solution

In the case that three lines L_1, L_2 and L_3 are all rays from P_0, three features construct a corner feature which is studied in literature [8]. See as Fig. 4, suppose the plane defined by O_C and l_1, l_2, l_3 are π_1, π_2, π_3 respectively. From P_0 make a plane which is perpendicular to O_CP_0, the intersection lines of this plane and π_1, π_2, π_3 are L_1', L_2', L_3' respectively; the angel between L_i and L_i' is β_i $(i = 1,2,3)$. From given conditions, it is easily to calculate the angles between the ray L_i' and L_j', suppose they are γ_{ij} $(i, j = 1,2,3)$. It is not difficult to prove the angles must satisfy the following equation system with respect to β_i.

$$\begin{cases} \tan\beta_1 \cdot \tan\beta_2 + \cos\gamma_{12} = \cos\alpha_{12} \cdot \sqrt{(1+\tan^2\beta_1)(1+\tan^2\beta_2)} \\ \tan\beta_1 \cdot \tan\beta_3 + \cos\gamma_{13} = \cos\alpha_{13} \cdot \sqrt{(1+\tan^2\beta_1)(1+\tan^2\beta_3)} \\ \tan\beta_2 \cdot \tan\beta_3 + \cos\gamma_{23} = \cos\alpha_{23} \cdot \sqrt{(1+\tan^2\beta_2)(1+\tan^2\beta_3)} \end{cases} . \tag{4}$$

Generally (4) is a nonlinear equation system, it can be solved by nonlinear methods. After we obtain the value of β_1, β_2 and β_3, we can get the direction of three rays L_1, L_2 and L_3 and then obtain the rotation parameters. From $|P_0P| = d$, we can obtain the translation vector.

In following two simple cases, we can obtain the closed form solutions of the problem.

1) Three rays are perpendicular to each other

In this case, $\alpha_{12} = \alpha_{13} = \alpha_{23} = 90^\circ$, then we have $\cos\alpha_{12} = \cos\alpha_{13} = \cos\alpha_{23} = 0$, (4) is simplified as

$$\begin{cases} \tan\beta_1 \cdot \tan\beta_2 + \cos\gamma_{12} = 0 \\ \tan\beta_1 \cdot \tan\beta_3 + \cos\gamma_{13} = 0 \\ \tan\beta_2 \cdot \tan\beta_3 + \cos\gamma_{23} = 0 \end{cases} . \tag{5}$$

It is easy to obtain two solutions of (5)

$$\begin{cases} \tan\beta_1 = \pm\sqrt{-\dfrac{\cos\gamma_{12}\cdot\cos\gamma_{13}}{\cos\gamma_{23}}} \\ \tan\beta_2 = -\dfrac{\cos\gamma_{12}}{\tan\beta_1} \\ \tan\beta_3 = -\dfrac{\cos\gamma_{13}}{\tan\beta_1} \end{cases} .$$

Thinking the direction relationship of three rays L_1, L_2 and L_3 is given, only one solution is possible solution in the above two, so in this case the problem has unique solution. The result is same as that in [7].

2) One ray is perpendicular to the other two rays

In this case, suppose $\alpha_{12} = \alpha_{13} = 90^\circ$, then we have $\cos\alpha_{12} = \cos\alpha_{13} = 0$, (4) is

$$\begin{cases} \tan\beta_1 \cdot \tan\beta_2 + \cos\gamma_{12} = 0 \\ \tan\beta_1 \cdot \tan\beta_3 + \cos\gamma_{13} = 0 \\ \tan\beta_2 \cdot \tan\beta_3 + \cos\gamma_{23} = \cos\alpha_{23}\cdot\sqrt{(1+\tan^2\beta_2)(1+\tan^2\beta_3)} \end{cases} . \tag{6}$$

Let

$$\begin{cases} a = \cos^2\gamma_{23} - \cos^2\alpha_{23} \\ b = 2\cos\gamma_{12}\cdot\cos\gamma_{13}\cdot\cos\gamma_{23} - \cos^2\alpha_{23}\cdot\cos^2\gamma_{12} - \cos^2\alpha_{23}\cdot\cos^2\gamma_{13} \\ c = \cos^2\gamma_{12}\cdot\cos^2\gamma_{13}\left(1-\cos^2\alpha_{23}\right) \end{cases} . \tag{7}$$

From (6), we can get following equation with respect to $\tan\beta_1$

$$a\cdot\tan^4\beta_1 + b\cdot\tan^2\beta_1 + c = 0 . \tag{8}$$

(8) is a quadratic equation with respect to $\tan^2\beta_1$. It is easy to know c in (7) is greater than 0, so if $a \le 0$, (8) has unique positive solution of $\tan^2\beta_1$, and we can get two supplementary solutions of β_1. Thinking the direction relationship of L_1, L_2 and L_3, only one is possible solution. If $a > 0$, (8) has two positive solutions of $\tan^2\beta_1$, and then we can get two pairs (total four) of supplementary solutions of β_1. Thinking the direction relationship of L_1, L_2 and L_3, the problem has two possible solutions.

Summarize the above discussion, we have the following theorem.

Theorem 2: Regarding the problem described in Fig. 4, the solution number of the problem depends on the relationship of the three rays and the location of the optical center and three features. If only one ray is perpendicular to other two rays, the solution number is two at most; if three rays are perpendicular to each other, the problem has unique solution. In above two cases, the problem has closed form solutions; in other cases it is not easy to simplify (4), the problem's solutions can only be obtained by nonlinear methods.

5 Conclusion

In this paper we discuss the pose determination problem in monocular vision using one point and two coplanar line features. By geometric analysis, we demonstrate that the problem's solution number depends on the location of the camera's optical center and the geometric relationship of the features. In some cases, the closed form solutions of the problem exist. It is a real time method to obtain the position and orientation parameters from simple features. Because the point and line mixed features widely exist in the real world, the results in this paper provide us a new pose measurement method in computer vision systems.

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